Formulae Sheet
Mathematics

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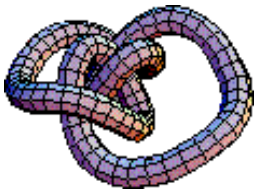

Instructor: Jorge R. Viramontes
Mathematics

1) $\sin t = \dfrac{opp}{hyp} = \dfrac{y}{r} = \dfrac{1}{\csc t}$

2) $\cos t = \dfrac{ady}{hyp} = \dfrac{x}{r} = \dfrac{1}{\sec t}$

3) $\tan t = \dfrac{opp}{ady} = \dfrac{y}{x} = \dfrac{1}{\cot t}$

4) $\tan t = \dfrac{\sin t}{\cos t}$

5) $\cot t = \dfrac{\cos t}{\sin t}$

6) $\sin^2 t + \cos^2 t = 1$

7) $\sec^2 t - \tan^2 t = 1$

8) $\csc^2 t - \cot^2 t = 1$

9) $\sin 2t = 2 \sin t \cos t$

10) $\cos 2t = 1 - 2\sin^2 t$

11) $\cos 2t = -1 + 2\cos^2 t$

12) $\cos 2t = \cos^2 t - \sin^2 t$

13) $\tan 2t = \dfrac{2\tan t}{1 - \tan^2 t}$

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14) $\sin t = \cos(\pi/2 - t)$

15) $\sec t = \csc(\pi/2 - t)$

16) $\tan t = \cot(\pi/2 - t)$

17) $\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$

18) $\sin(x \pm y) = \sin x \cos y \pm \sin y \cos x$

19) $\tan(x \pm y) = \dfrac{\tan x \pm \tan y}{1 \mp \tan x \tan y}$

20) $\cos(-t) = \cos t$

21) $\sin(-t) = -\sin t$

22) $\tan(-t) = -\tan t$

23) $\cos(t + 2\pi n) = \cos t \quad \text{where } n \in \mathbb{N}$

24) $\sin(t + 2\pi n) = \sin t \quad \text{where } n \in \mathbb{N}$

25) $\tan(t + \pi n) = \tan t \quad \text{where } n \in \mathbb{N}$

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26) $\sin^2 t = \dfrac{1 - \cos 2t}{2}$

27) $\cos^2 t = \dfrac{1 + \cos 2t}{2}$

28) $\tan^2 t = \dfrac{1 - \cos 2t}{1 + \cos 2t}$

29) $\sin t/2 = \pm\sqrt{\dfrac{1 - \cos t}{2}}$

30) $\cos t/2 = \pm\sqrt{\dfrac{1 + \cos t}{2}}$

31) $\tan t/2 = \pm\sqrt{\dfrac{1 - \cos t}{1 + \cos t}}$

32) $\dfrac{\pi}{2} - \arctan t = \arctan \dfrac{1}{t}$

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33) $\cos x + \cos y = 2\cos\left(\dfrac{x+y}{2}\right)\cos\left(\dfrac{x-y}{2}\right)$

34) $\cos x - \cos y = -2\sin\left(\dfrac{x+y}{2}\right)\sin\left(\dfrac{x-y}{2}\right)$

35) $\sin x + \sin y = 2\sin\left(\dfrac{x+y}{2}\right)\cos\left(\dfrac{x-y}{2}\right)$

36) $\sin x - \sin y = 2\cos\left(\dfrac{x+y}{2}\right)\sin\left(\dfrac{x-y}{2}\right)$

37) $\sin x \sin y = \dfrac{1}{2}\left[\cos(x-y) - \cos(x+y)\right]$

38) $\cos x \cos y = \dfrac{1}{2}\left[\cos(x-y) + \cos(x+y)\right]$

39) $\sin x \cos y = \dfrac{1}{2}\left[\sin(x+y) + \sin(x-y)\right]$

40) $\cos x \sin y = \dfrac{1}{2}\left[\sin(x+y) - \sin(x-y)\right]$

41) $s = r\theta$

41) $v = \omega r$

42) $A = \frac{1}{2}r^2\theta$

43) $v = s/t$

44) $\omega = \theta/t$

44) $c^2 = a^2 + b^2$

45) $\ln a + \ln b = \ln(a \cdot b)$

46) $\ln a - \ln b = \ln(a/b)$

47) $\ln b^t = t \ln a$

48) $c^2 = a^2 + b^2 - 2ab\cos C$

49) $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$

50) $V\left(\frac{-b}{2a}, f(-b/2a)\right)$

$\sin 0° = 0$

$\cos 0° = 1$

$\tan 0° = 0$

$\sin 30° = \frac{1}{2} = 0.5$

$\cos 30° = \frac{\sqrt{3}}{2} = 0.8660...$

$\tan 30° = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3} = 0.57735...$

$\sin 45° = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2} = 0.7071...$

$\cos 45° = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2} = 0.7071...$

$\tan 45° = 1$

$\sin 60° = \frac{\sqrt{3}}{2} = 0.8660...$

$\cos 60° = \frac{1}{2} = 0.5$

$\tan 60° = \frac{\sqrt{3}}{1} = 1.73205080...$

$\sin 90° = 1$

$\cos 90° = 0$

$\tan 90° = +\infty$

$\sin 180° = 0$

$\cos 180° = -1$

$\tan 180° = 0$

$\sin 270° = -1$

$\cos 270° = 0$

$\tan 270° = -\infty$

$\sin 360° = 0$

$\cos 360° = 1$

$\tan 360° = 0$

$1\ rad = \frac{180}{\pi} = 57.295779°\ldots$

$2\pi\ rad = 360°$

$\pi\ rad = 180°$

$\frac{\pi}{2}\ rad = 90°$

$\frac{\pi}{3}\ rad = 60°$

$\frac{\pi}{6}\ rad = 30°$

$\frac{\pi}{4}\ rad = 45°$

$\frac{3}{2}\pi\ rad = 270°$

$\frac{3}{4}\pi\ rad = 135°$

$a_1 \sin(Bt) + a_2 \cos(Bt) = A\sin(Bt + \phi)$

where $A = \sqrt{a_1^2 + a_2^2}$

and $\phi = \tan^{-1}\frac{a_2}{a_1}$

$\sinh t = \frac{1}{\operatorname{csch} t}$

$\sinh t = \frac{e^t - e^{-t}}{2}$

$\cosh^2 t - \sinh^2 t = 1$

$\cosh t = \frac{1}{\operatorname{sech} t}$

$\cosh t = \frac{e^t + e^{-t}}{2}$

$\operatorname{sech}^2 t + \tanh^2 t = 1$

$\tanh t = \frac{1}{\coth t}$

$\tanh t = \frac{\sinh t}{\cosh t}$

$\coth^2 t - \operatorname{csch}^2 t = 1$

$e^{\pm it} = \cos t \pm i \sin t$

$\sin t = \frac{e^{it} - e^{-it}}{2i}$

$\cos t = \frac{e^{it} + e^{-it}}{2}$

Limit Properties. Let f be a function which is defined at every number in some open interval containing a, except possibly at the number a itself. The limit of $f(x)$ as x approaches a is L, written as

$$\lim_{x\to a} f(x) = L,$$

iff, $\forall \epsilon > 0$, $\exists \delta > 0 \ni$, If $0 < |x - a| < \delta$, Then $|f(x) - L| < \epsilon$

If a, b, n, are real numbers, and f and g are functions, then

1) $\lim_{x\to a} b = b$

2) $\lim_{x\to a} x = a$

3) $\lim_{x\to a}[bf(x)] = b[\lim_{x\to a} f(x)]$

4) $\lim_{x\to a}[f(x) \pm g(x)] = \lim_{x\to a} f(x) \pm \lim_{x\to a} g(x)$

5) $\lim_{x\to a}[f(x) \cdot g(x)] = \lim_{x\to a} f(x) \cdot \lim_{x\to a} g(x)$

6) $\lim_{x\to a} \dfrac{f(x)}{g(x)} = \dfrac{\lim_{x\to a} f(x)}{\lim_{x\to a} g(x)}$ and $\lim_{x\to a} g(x) \neq 0$

7) $\lim_{x\to a}[f(x)]^n = [\lim_{x\to a} f(x)]^n = A^n$

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8) If f is a polynomial function, then

$$\lim_{x\to a} f(x) = f(a)$$

9) If f and g are functions such that $\lim_{x\to a} g(x) = L$, and $\lim_{x\to L} f(x) = f(L)$, then

$$\lim_{x\to a} f(g(x)) = f\left(\lim_{x\to a} g(x)\right) = f(L)$$

10) Let f and g be two functions such that $f(x) = g(x)$ for all $x \neq a$ in an open interval containing a, then

$$\lim_{x\to a} f(x) = \lim_{x\to a} g(x)$$

11) If $h(x) \leq f(x) \leq g(x)$ for all x in an open interval containing a, except possibly at a itself, and if

$$\lim_{x\to a} h(x) = L = \lim_{x\to a} g(x)$$

then $\lim_{x\to a} f(x)$ exists and is equal to L.

12) Limits of Trigonometric Functions. Let a be a real number in the domain of the given trigonometric function.

$$\lim_{x\to a} \sin x = \sin a \quad \lim_{x\to a} \cos x = \cos a \quad \lim_{x\to a} \tan x = \tan a \quad \lim_{x\to 0} \frac{\sin x}{x} = 1 \quad \lim_{x\to 0} \frac{1 - \cos x}{x} = 0$$

Derivatives. If $u = u(x)$, $v = v(x)$, and $w = w(x)$, then

1) $D_x c = 0$

2) $D_x x = 1$

3) $D_x u^n = n u^{n-1} D_x u$

4) $D_x(u \pm v) = D_x u \pm D_x v$

5) $D_x(c \cdot u) = c D_x u$

6) $D_x \left(\frac{u}{v}\right) = \frac{v D_x u - u D_x v}{v^2}$

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7) $D_x f(u) = D_u f(u) \cdot D_x u$

8) $D_x e^u = e^u D_x u$

9) $D_x a^u = a^u \ln a D_x u$

10) $D_x \log_a u = \frac{1}{u} \log_a e \; D_x u$

11) $D_x u^v = v u^{v-1} D_x u + u^v \ln u D_x v$

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12) $D_x \sin u = \cos u D_x u$

13) $D_x \cos u = -\sin u D_x u$

14) $D_x \tan u = \sec^2 u D_x u$

15) $D_x \sec u = \sec u \tan u D_x u$

16) $D_x \csc u = -\csc u \cot u D_x u$

17) $D_x \cot u = -\csc^2 u D_x u$

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18) $D_x \arcsin u = \frac{1}{\sqrt{1-u^2}} D_x u$

19) $D_x \arccos u = \frac{-1}{\sqrt{1-u^2}} D_x u$

20) $D_x \arctan u = \frac{1}{1+u^2} D_x u$

21) $D_x \operatorname{arccot} u = \frac{-1}{1+u^2} D_x u$

22) $D_x \operatorname{arcsec} u = \frac{1}{u\sqrt{u^2-1}} D_x u$

23) $D_x \operatorname{arccsc} u = \frac{-1}{u\sqrt{u^2-1}} D_x u$

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24) $D_x \sinh u = \cosh u D_x u$

25) $D_x \cosh u = \sinh u D_x u$

26) $D_x \tanh u = \operatorname{sech}^2 u D_x u$

27) $D_x \coth u = -\operatorname{csch}^2 u D_x u$

28) $D_x \operatorname{sech} u = -\operatorname{sech} u \tanh u D_x u$

29) $D_x \operatorname{csch} u = -\operatorname{csch} u \coth u D_x u$

Integrals. If $u = u(x)$, $v = v(x)$, and $c = constant$, then

1) $\int cf(x)\,dx = c\int f(x)\,dx$

2) $\int (u \pm v)\,dx = \int u\,dx \pm \int v\,dx$

3) $\int dx = x + c$

4) $\int f(ax)\,dx = \frac{1}{a}\int f(u)\,du$

5) $\int u^n\,du = \frac{u^{n+1}}{n+1} + c$

6) $\int u\,dv = uv - \int v\,du$

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7) $\int e^u\,du = e^u + c$

8) $\int \frac{dx}{x} = \ln|x| + c$

9) $\int a^u\,du = \frac{a^u}{\ln a} + c$

10) $\int \sin u\,du = -\cos u + c$

11) $\int \cos u\,du = \sin u + c$

12) $\int \tan u\,du = \ln|\sec u| + c$

13) $\int \sec u\,du = \ln|\sec u + \tan u| + c$

14) $\int \csc u\,du = \ln|\csc u - \cot u| + c$

15) $\int \cot u\,du = -\ln|\csc u| + c$

16) $\int \sec^2 u\,du = \tan u + c$

17) $\int \csc^2 u\,du = -\cot u + c$

18) $\int \sec u \tan u\,du = \sec u + c$

19) $\int \csc u \cot u\,du = -\csc u + c$

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20) $\int \frac{du}{u^2 + a^2} = \frac{1}{a}\arctan\frac{u}{a} + c$

21) $\int \frac{du}{u^2 - a^2} = \frac{1}{2a}\ln\frac{u-a}{u+a} + c$

22) $\int \frac{du}{a^2 - u^2} = \frac{1}{2a}\ln\frac{a+u}{a-u} + c$

23) $\int \frac{du}{\sqrt{a^2 - u^2}} = \arcsin\frac{u}{a} + c$

24) $\int \frac{du}{\sqrt{u^2 \pm a^2}} = \ln(u + \sqrt{u^2 \pm a^2}) + c$

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25) $\int \sinh u\,du = \cosh u + c$

26) $\int \cosh u\,du = \sinh u + c$

27) $\int \tanh u\,du = \ln(\cosh u) + c$

28) $\int \operatorname{sech} u\,du = \arcsin(\tanh u) + c$

29) $\int \operatorname{csch} u\,du = \ln(\tanh\frac{u}{2}) + c$

30) $\int \coth u\,du = \ln(\sinh u) + c$

31) $\int \operatorname{sech}^2 u\,du = \tanh u + c$

32) $\int \operatorname{csch}^2 u\,du = -\coth u + c$

33) $\int \sqrt{a^2-u^2}du = \frac{u}{2}\sqrt{a^2-u^2} + \frac{a^2}{2}\arcsin\frac{u}{a} + c$ 34) $\int \sqrt{u^2 \pm a^2}du = \frac{u}{2}\sqrt{u^2 \pm a^2} + \frac{a^2}{2}\ln(u + \sqrt{u^2 \pm a^2}) + c$

35) $\int u^n e^{au}\, du = \frac{u^n e^{au}}{a} - \frac{n}{a}\int u^{n-1}e^{au}\, du$

36) $\int e^{au}\sin nu\, du = \frac{e^{au}}{a^2+n^2}\left[a\sin nu - n\cos nu\right]$

37) $\int e^{au}\cos nu\, du = \frac{e^{au}}{a^2+n^2}\left[n\sin nu + a\cos nu\right]$

38) $\int f^{(n)}g\, du = f^{(n-1)}g - f^{(n-2)}g' + f^{(n-3)}g'' - \cdots(-1)^n\int fg^{(n)}\, du$

$a^n - b^n = (a-b)(a^{n-1} + a^{n-2}b + a^{n-3}b^2 + \cdots + ab^{n-2} + b^{n-1})\ n \in N$

$a^n + b^n = (a+b)(a^{n-1} - a^{n-2}b + a^{n-3}b^2 - \cdots - ab^{n-2} + b^{n-1})\ n \in \mathbb{N} \cap \{2,4,\ldots\}$

$e = 2.71\ 82\ 81\ 82\ 84\ 59\ 04\ 52\ 35\ 36\ 02\ 87...$

$\pi = 3.14\ 15\ 92\ 65\ 35\ 89\ 79\ 32\ 38\ 46\ 26\ 43\ 38\ 32\ 79\ 50\ 28\ 84\ 19\ 71\ 69\ 39\ 93\ 75\ 10\ 58\ 20\ 97\ 49\ 44\ 59\ 23\ 07\ 81\ 64\ 06\ 28\ 62$
$08\ 99\ 86\ 28\ 03\ 48\ 25\ 34\ 21\ 17\ 06\ 79\ 82\ 14\ 80\ 86\ 51\ 32\ 82\ 30\ 66\ 47\ 09\ 38\ 44\ 60\ 95\ 50\ 58\ 22\ 31\ 72\ 53\ 59\ 40\ 81\ 28\ 48\ 11\ 17$
$45\ 02\ 84\ 10\ 27\ 01\ 93\ 85\ 21\ 10\ 55\ 59\ 64\ 46\ 22\ 94\ 89\ 54\ 93\ 03\ 81\ 96\ldots$

Número de Eddington=

$15,747,724,136,275,002,577,605,653,961,181,555,468,044,717,914,527,116,709,366,231,425,076,185,631,031,296$

$1rad = \frac{180}{\pi} = 57.295779^\circ\ldots$ $\sqrt{2} = 1.41421356\ldots$ $\sqrt{3} = 1.73205080\ldots$

$\sqrt{5} = 2.23606797\ldots$ $\sqrt{6} = 2.4494897\ldots$ $\sqrt{7} = 2.64575131\ldots$

$\sqrt{8} = 2.8284\ldots$ $\sqrt{11} = 3.3166\ldots$ $e^{\pi} = 23.14069263\ldots$

$1/2 = 0.5$ $1/3 = 0.33\ldots$ $1/4 = 0.25$

$1/5 = 0.2$ $1/6 = 0.16\ldots$ $1/7 = 0.142857\ldots$

$1/8 = 0.125$ $1/9 = 0.11\ldots$