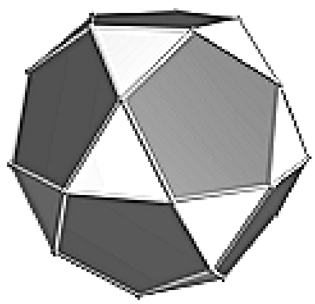
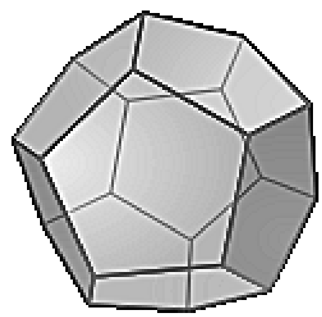
Instructor: Jorge R. Viramontes
MATHEMATICS 3309

16.1 Shoeboxes Have Faces and Nets!

Definition. Polyhedra. A **polyhedron** (plural polyhedra or polyhedrons) is often defined as a geometric object with flat faces and straight edges (the word polyhedron comes from the Classical Greek $\pi o\lambda\upsilon\epsilon o\rho o\nu$, from poly, "many," edron, "face").
In geometry, a polyhedron is traditionally a three-dimensional shape that is made up of a finite number of polygonal faces which are parts of planes; the faces meet in pairs along edges which are straight-line segments, and the edges meet in points called vertices. Cubes, prisms and pyramids are examples of polyhedra. The polyhedron surrounds a bounded volume in three-dimensional space; sometimes this interior volume is considered to be part of the polyhedron, sometimes only the surface is considered, and occasionally only the skeleton of edges.





Problem 1. *Draw a shoebox.*

Definition. From the previous problem, the flat parts of the shoebox are examples of two dimensional (2D) or planar regions, or sometimes called **faces**. The corner points of the shoebox are called **vertices** (singular: vertex), and the line segments where two faces meet are called **edges**.

Problem 2. *Draw a cube and a shoebox using isometric dot paper.*

Definition. Nets. For manufacturing purposes, boxes may be made from patterns on flat material, and then folded up and taped. Viceversa, some edges of a box can be cut so that the box can be unfolded to lie flat in one connected piece. Unfolded versions of a 3D shape are called **nets**.

Problem 3. *From figure 3, page 367, which of the followings nets gives a shoebox without its lid, if it is folded up and taped?*

Using the regular format for homework: Page 368 all problems 1-7

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16.2 Introduction to Polyhedra.

Problem 1. *Write your name inside the outlines for shapes A-M (their nets), found in Appendix G. Before cutting out the shape, try to imagine what the final, folded-up shape will look like. Cut around the outside of each shape, fold on the other line segments (a ruler helps make sharp folds; fold so the shape letter and your name will show), and then tape the shape together. Find a small container so that you can bring the shapes to class without damaging them.*

Problem 2. *How would you sort shapes A-K into different sets? Write down your word description for the sets.*

Remark. Shapes A, D, G, and I are called **pyramids**, one of the faces of the pyramid is called the **base**.

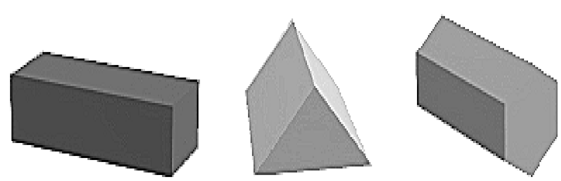
Problem 3. *Base on the previous remark,*

a. *Propose a definition for pyramid.*
b. *Identify the base of each pyramid.*
c. *What is the shape of the base called?*

Remark. Shapes B, C, E, and F are called **prisms**, two of the faces of the prism are called the **bases**.

Problem 4. *Base on the previous remark,*

a. *Propose a definition for prism.*
b. *Identify the bases of each prism.*
c. *How are they related to each other?*



Problem 5. *Find all the pyramids there are in shapes A-K. Count the number of F faces, the number of V of vertices, and the number of E edges of each pyramid. Organize the data in table and look for patterns or relationships.*

Problem 6. *Find similar counts for a hexagonal pyramid, a 100-gonal pyramid, and n-gonal pyramid, and an $(x+y)$-gonal pyramid and add these counts to your data. Do the patterns or relationships hold for these pyramids also?*

Problem 7. *One relationship that you may have overlooked relates all three quantities V, F, and E, along with possibly a specific number. Try to find this relationship.*

Problem 8. *Find all the prisms there are in shapes A-K. Count the number of F faces, the number of V of vertices, and the number of E edges of each prism. Organize the data in table and look for patterns or relationships.*

Problem 9. *Find similar counts for a hexagonal prism, a 100-gonal prism, and n-gonal prism, and add these counts to your data. Do the patterns or relationships hold for these prisms also?*

Problem 10. *One relationship that you may have overlooked relates all three quantities V, F, and E, along with possibly a specific number. Try to find this relationship.*

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16.3 Representing and Visualizing Polyhedra.

Proposition. *There are four different representations of a polyhedron:*

a) *Physical model (3D object)*
b) *Word (verbal)*
c) *Net (unfolded flat)*
d) *Drawing (2D picture)*

We may **translate** *among the different representations of a polyhedron.*

Problem 1. *Unfolding a Cube. Using a cube or just thinking about a cube, draw a net for it.*

Remark. There are different kinds of drawing: *perspective drawing*, and mathematics (no perspective).

Problem 2. *Draw a cube using the two kinds of drawing (with hidden and not hidden edges).*

Problem 3. *Using the shapes A-D draw a net for them.*

Problem 4. *Using the nets E-K draw the corresponding shape (use* **hidden edges***).*

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16.4 Congruent Polyhedra.

Definition. Replacement parts for a car or computer, cookies cut from a cookie cutter, or blouses made from the same pattern. Every object in one of these groups mentioned is expected to be exactly the same size and shape. They can be regarded as copies of one another. As you may recall, the technical word for this "exactly the same size and shape" idea is **congruence**.

Using the regular format for homework: Page 384 all problems 1-9

Definition. A **polygon** is a close figure made up line segments joined end to end without crossing over. The line segments are called **sides** of the polygon, and the endpoints are the **vertices** of the polygon.

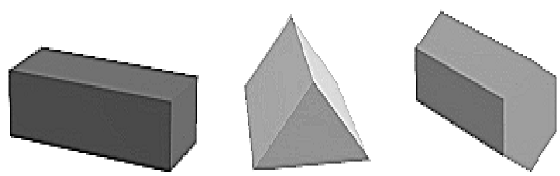
Definition. A **quadrilateral** is a polygon having 4 sides.

Definition. A **parallelogram** is a quadrilateral with both pairs of opposite sides parallel.

Definition. A **polyhedra** is a three-dimensional object made up of a finite number of polygonal faces; the faces meet in pairs along edges which are straight-line segments, and the edges meet in points called vertices.

Definition. A **pyramid** is any three-dimensional polyhedron where the faces other than the base are triangular and converge on one point, called the **apex**. The base of a pyramid can be any polygon but is typically a square, leading to four non-base faces. A pyramid is said to be regular if its base is a regular polygon and its upper faces are congruent isosceles triangles.

Definition. A **prism** is a polyhedron, with two parallel faces called bases. The other faces are always parallelograms. The prism is named by the shape of its base. Here are some types of prisms



Proposition. *The* **Euler's formula por polyhedra**. $V + F = E + 2$, *where* V *is the number of vertices of a given polyhedron,* F *is the number of faces, and* E *is the number of edges.*

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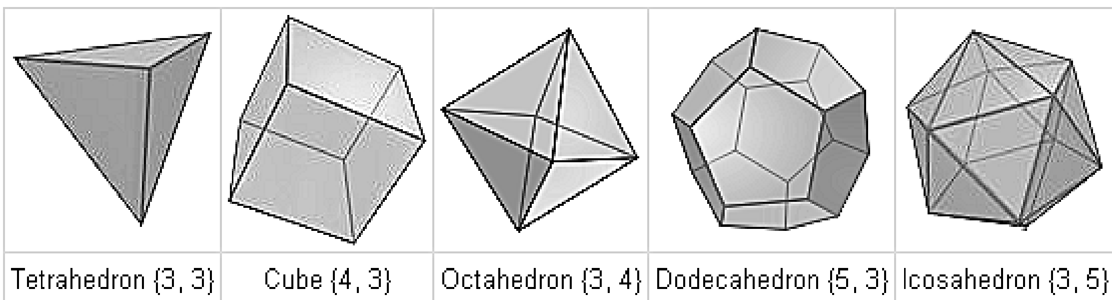
16.5 Some Special Polyhedra.

Definition. A **regular polyhedron**, or **Platonic solid**, is a polyhedron all of whose faces are congruent regular polygonal regions of one particular type, with the same arrangement of polygonal regions at each vertex. Regular polygons have all their sides the same lenght and all their angles the same size. A regular polyhedron is identified by its **Schläfli** symbol of the form $\{n, m\}$, where n is the number of sides of each face and m the number of faces meeting at each vertex.



A regular **tetrahedron** has four faces, each an equilateral triangular region. A regular **hexahedron** has six faces, each a square region (cube). A regular **octahedron** has eight faces, each an equilateral triangular region. Is there a regular polyhedron for every number of faces or at least for every even number of faces? The answer is "No." There are only five types of regular polyhedra. The other two types are the regular **dodecahedron** (dodeca means 12) and the regular **icosahedron** (icosa means 20).

Problem 1. *Build a dodecahedron and an icosahedron using the nets at the end of the book.*

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17.1 Review of Polygon Vocabulary.

Definition. A **polygon** is a closed planar figure made up of line segments joined end to end, with no crossing or re-use of endpoints. See example in page 396.

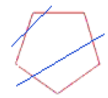
Definition. A polygon consist of just the points on its line segments. The points inside a polygon make up its polygonal region. An **equiangular** polygon is a polygon whose angles all have the same size. An **equilateral** polygon is a polygon whose sides all have the same length. Some polygons can be both equiangular and equilateral; in this case, the polygons are called **regular polygons**. See example in page 397.
Triangles can be classified by angles and by sides. Acute triangle has each angle measuring less than 90^o in size; a **right triangle** has an angle that is 90^o in size; and **obtuse triangle** has an angle that is more than 90^o in size. The **scalene triangle** all have different lenghts; the **isosceles traingle** two sides of the triangle have the same size; and the **equilateral triangle** if all three sides have the same lenght.

Definition. To denote a point we use capital letters A, B,.... Line segments with endpoints at P and Q: $\overline{PQ}$. Ray starting at C and going through D: $\overrightarrow{CD}$. Line trough D and E: $\overleftrightarrow{DE}$. Polygon with vertices F, G, H, and I: FGHI. Line segments and angles may also be indicated with small letters, which may also mean their lengths (and angle sizes). The end points of a line segment may be highlighted by dots. Angles are named either by naming just the vertex, or for clarity, naming a point on one side, the vertex, and then a point on the other side: $\angle A$, $\angle ABC$ respectively.

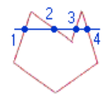
Definition. Vocabulary for Angles.

- A *straight angle* measures 180^o
- A *right angle* measures 90^o
- An *acute angle* measures less than 90^o
- An *obtuse angle* measures more than 90^o
- *Complementary angles* are two angles whose sum is 90^o. Each angle is called complement of the other.
- *Supplementary angles* are two angles whose sum is 180^o. Each angle is called supplement of the other.
- Two angles are *adjacent angles* if they have the same vertex and a common side between them.
- Two angles are *congruent* if they have the same measure.
- An *exterior* angle of a polygon is formed by the extension of one side of the polygon through a vertex of the polygon and the other side of the polygon through that vertex. We refer to an angle inside a polygon as **an angle** of the polygon, or an **interior** angle of the polygon.

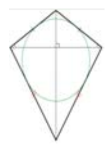
- A polygon is **convex**, if every straight line drawn will cross the polygon in at most two sides. Every interior angle is less than 180^o.



- A polygon is **concave**, if at least one straight line drawn will cross the polygon in more than two sides. At least one interior angle is more than 180^o.



- A **kite**, is a quadrilateral with two disjoint pairs of congruent adjacent sides, in contrast to a parallelogram, where the congruent sides are opposite.



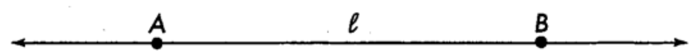
Using the regular format for homework: Page 400 all odd problems 1-21

1. Geometry.

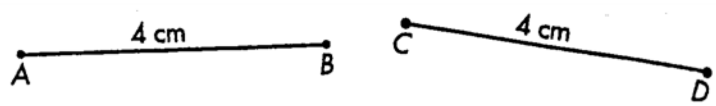
1.1. Points, Lines, Planes, and Angles.

Definition. .

- A point indicates a position. It has no length, no width, and no thickness.
- A line is a set of points. It may be considered as having only one dimension: length.
- The part of the line between two points on the line is called a *line segment*.

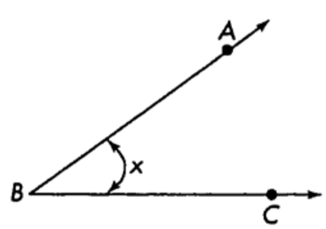


- When two segments have the same measure, they are called *congruent*.

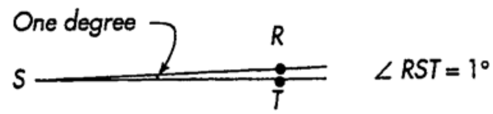


Congruent segments

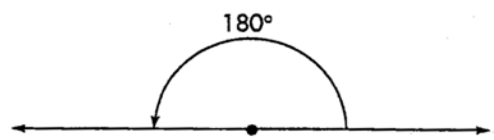
- A plane is a set of points that form a flat surface.
- An angle is the figure formed by two straight lines radiating from the same point. The two rays are called *sides* of the angle, and their common point is called the *vertex*. The symbol for angle is $\angle$. The figure shows $\angle x$ or $\angle\, ABC$.



- The *degree* is a unit of measure for angles. One degree is quite small, the symbol for degree is ^{o}.

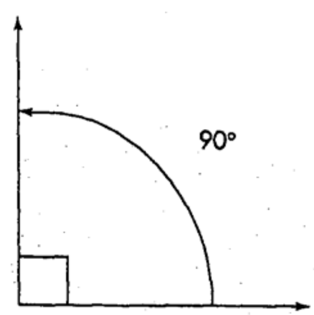


- A *straight angle* measures 180^o



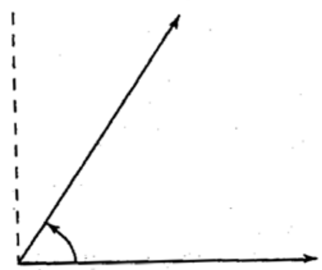
Straight angle

- A *right angle* measures 90^o



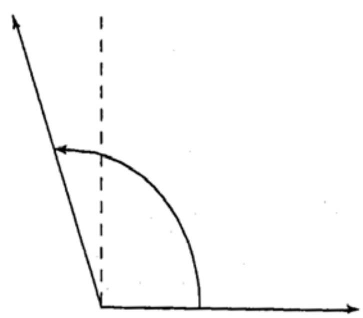
Right angle

- An *acute angle* measures less than 90^o



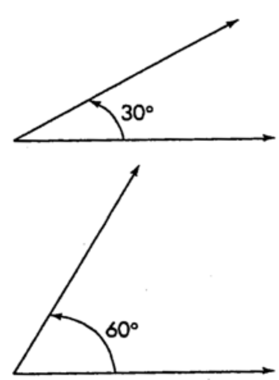
Acute angle

- An *obtuse angle* measures more than 90^o



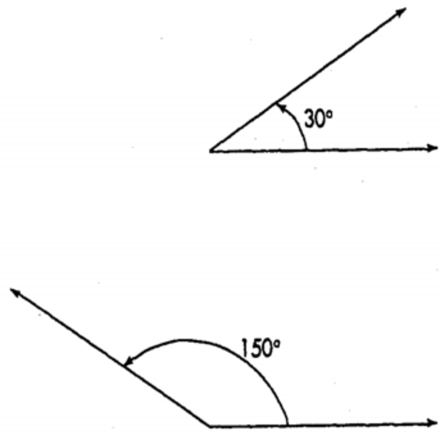
Obtuse angle

- *Complementary angles* are two angles whose sum is 90^o. Each angle is called complement of the other.



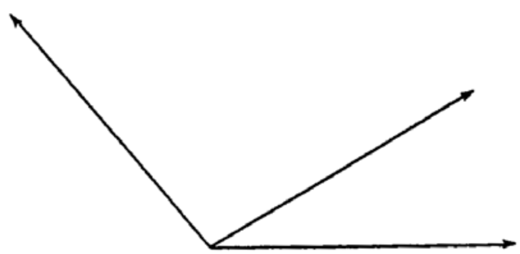
Complementary angles

- *Supplementary angles* are two angles whose sum is 180^o. Each angle is called supplement of the other.



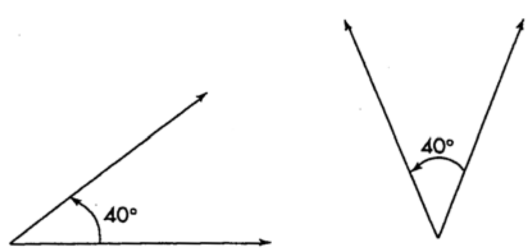
Supplementary angles

- Two angles are *adjacent angles* if they have the same vertex and a common side between them.



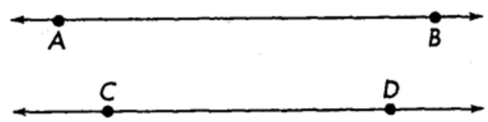
Adjacent angles

- Two angles are *congruent* if they have the same measure.



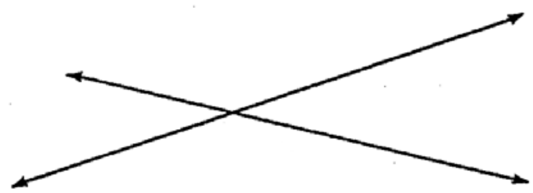
Congruent angles

- Lines in a plane can be parallel or intersecting. *Parallel lines* never meet. The distance between them is always the same.



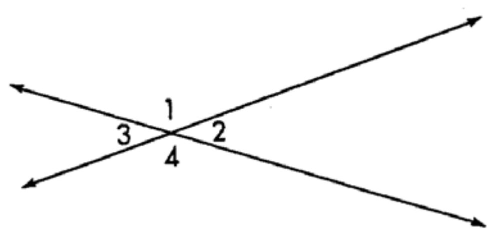
Parallel lines

- Intersecting lines cross at a point in the same plane.



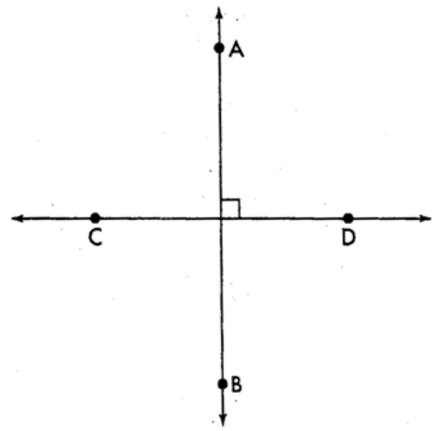
Intersecting lines

- When two lines intersect, the pairs or nonadjacent angles that are formed are called *vertical angles*.



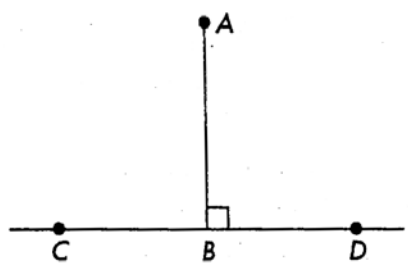
Vertical angles

- Two lines that intersect so that a right angle is formed at their intersection are said to be *perpendicular* to each other.



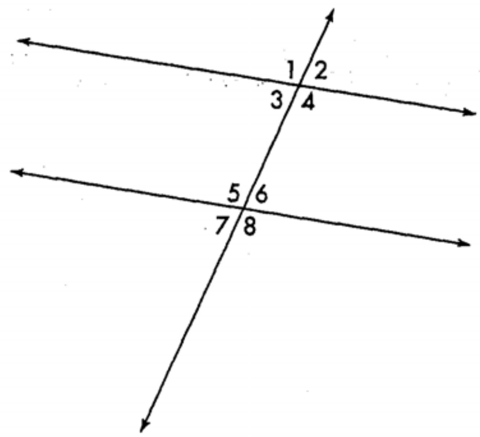
Perpendicular lines

- The *distance* from a point to a line is the length of the perpendicular line segment from the point to the line.



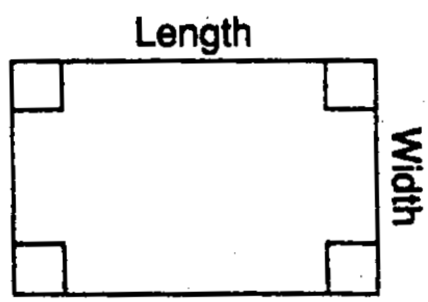
Distance

- A *transversal* is a line that intersects two or more lines. When this happen, eight angles are formed. This angles are named as follows.
  Alternate interior angles, such as $\angle 3$ and $\angle 6$ or $\angle 4$ and $\angle 5$.
  Alternate exterior angles, such as $\angle 1$ and $\angle 8$ or $\angle 2$ and $\angle 7$.
  Corresponding angles, such as $\angle 1$ and $\angle 5$, $\angle 2$ and $\angle 6$, $\angle 3$ and $\angle 7$, or $\angle 4$ and $\angle 8$.



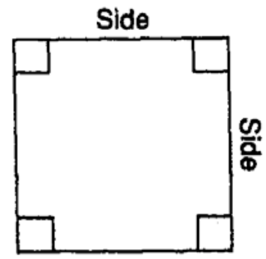
Transversal

- A *rectangle* is a closed, four-side figure that has 4 right angles. It has two dimensions: length and width.



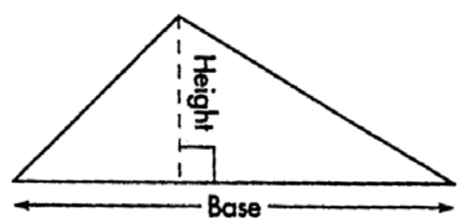
Rectangle

- A *square* is a rectangle with four equal sides. Its two dimensions, length and width, are equal.

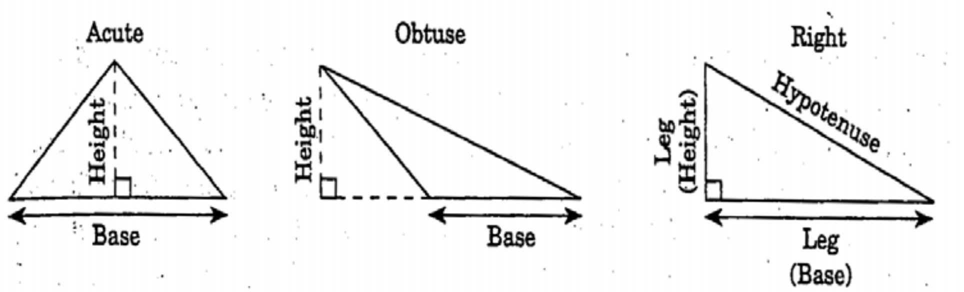


Square

- A *triangle* is a closed, three-sided plane figure. It has two dimensions: base, the length of one side, and height, the distance from the opposite vertex to the base. (the height and the base meet at a right angle).

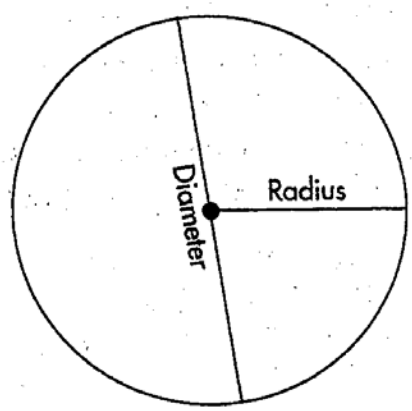


Triangle



Acute, obtuse, and right triangles

- A *circle* is a closed plane figure all points of which are the same distance from a point within called center. The *diameter* of a circle is a line segment through the center of the circle with endpoints on the circle. The diameter of a circle is twice as long as the radius.



Circle

1.2. **Solving Perimeter Problems.**

- Formula for the perimeter of a rectangle $P = 2l + 2w$, where l is the length and w is the width.

- Formula for the perimeter of a square $P = 4l$, where l is the length.

- Formula for the perimeter of a triangle $P = l1 + l2 + l3$, where $l1$, $l2$, $l3$ is the length of each side of the triangle.

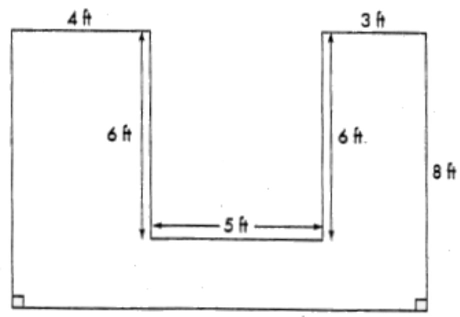
- Formula for the perimeter of a circle $P = 2\pi r$, where r is the radius of the circle.

Problem. *A rectangle has a lenght of 36 cm and a perimeter of 116 cm. Find the width.*

Problem. *Find the perimeter of a square that is 40 m long on each side.*

Problem. *A triangular-shaped garden is to be enclosed with a fance. One side of the garden is 20 ft long, a second side is 15 ft, and the third side is 28 feet. How many feet of fence will enclose the garden?*

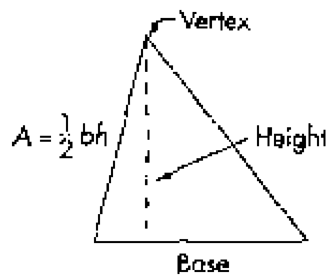
Problem. *Find the perimeter of the following figure.*



Problem. *Find the perimeter of the circle with diameter equal to 8 cm.*

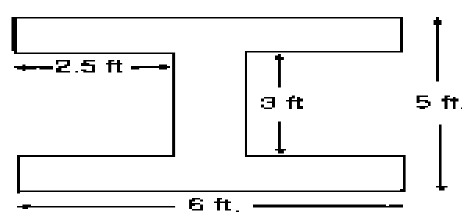
1.3. **Solving Area Problems.**

- Formula for the area of a rectangle $A = l \cdot w$, where l is the length and w is the width.
- Formula for the area of a square $A = l^2$, where l is the length.
- Formula for the area of a triangle $A = \dfrac{b \cdot h}{2}$, where b is the base, h is the height.



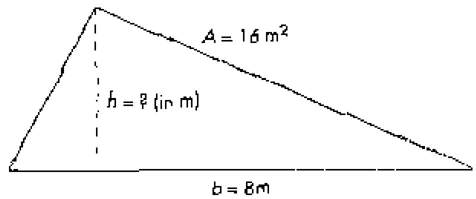
- Formula for the area of a circle $A = \pi r^2$, where r is the radius of the circle.

Problem. *Find the area of the following figure.*

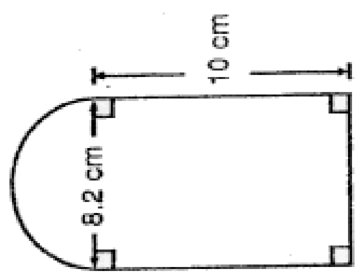


Problem. *Jorge & Alonso want to carpet their den, which is 25 feet by 17 feet. How many square feet of carpet will they need? Assume 5% of waste.*

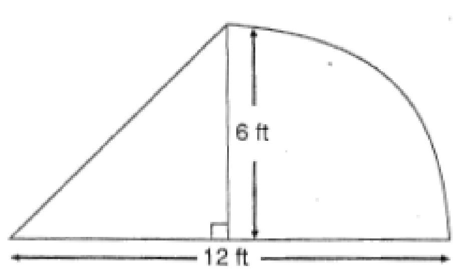
Problem. *Find the the height of the triangle shown below.*



Problem. *Find the area of the figure in the diagram. Use* $\pi \cong 3.14$ *(Consider the left end as a half of a circle).*

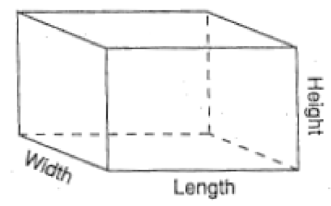


Problem. *Find the area of the figure in the diagram. Use* $\pi \cong 3.14$ *(Consider the right end as a fourth of a circle).*



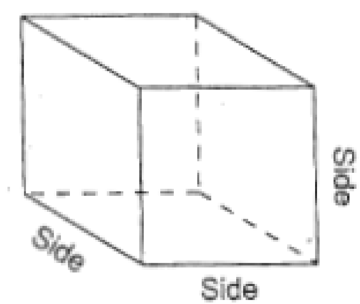
1.4. **Solving Surface Area Problems.**

- Formula for the Surface area of a rectangular solid $SA = 2(lw + lh + wh)$, where l is the length, w is the width, and h is the height.



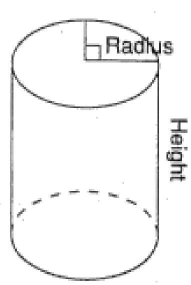
Rectangular solid

- Formula for the Surface area of a cube $SA = 6l^2$, where l is the measure of one side.



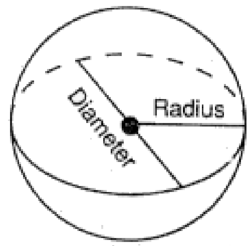
Cube

- Formula for the Surface area of a right circular cylinder $SA = 2\pi r^2 + 2\pi rh$, where r is the radius of the cross section and h is the height of the circular cylinder.



Cylinder

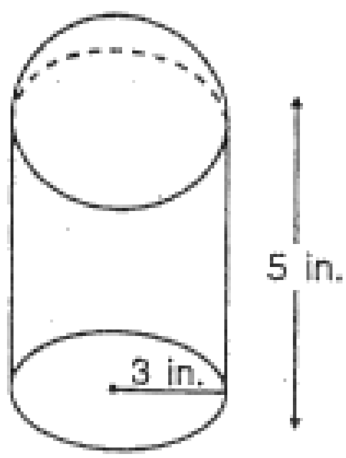
- Formula for the Surface area of a sphere $SA = 4\pi r^2$, where r is the radius of the circle.



Sphere

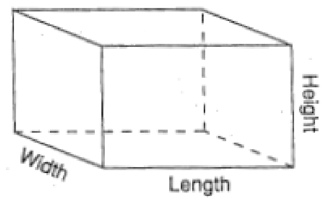
Problem. *How many square feet of paper are needed to cover the outside of a cube-shaped object that is 4 feet on a side? Assume 5% of waste.*

Problem. *Find the surface area of the following figure (consider the dome a hemisphere).*



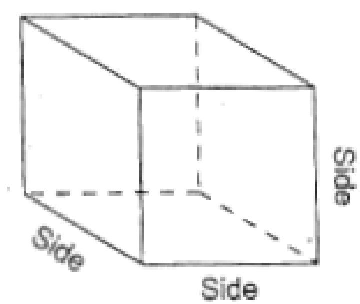
1.5. **Solving Volume Problems.**

- Formula for the Volume of a rectangular solid $V = lwh$, where l is the length, w is the width, and h is the height.



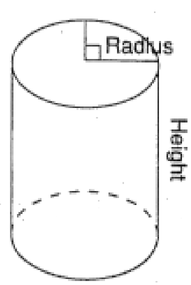
Rectangular solid

- Formula for the Volume of a cube $V = l^3$, where l is the measure of one side.



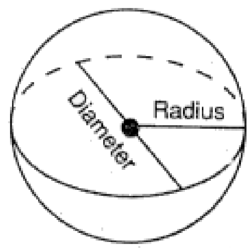
Cube

- Formula for the Volume of a right circular cylinder $V = \pi r^2 h$, where r is the radius of the cross section and h is the height of the circular cylinder.



Cylinder

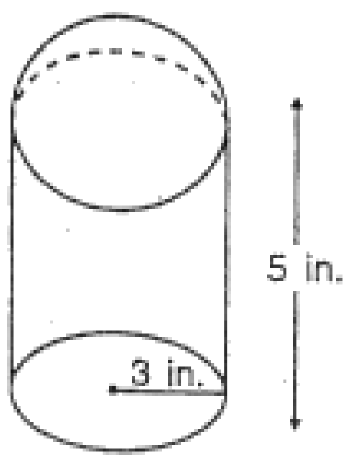
- Formula for the Volume of a sphere $V = \frac{4}{3}\pi r^3$, where r is the radius of the sphere.



Sphere

Problem. *A classroom is 40 feet long, 25 feet wide, and 12 feet high. How many cubic feet of space are in the room?*

Problem. *Find the volume of a paperweight that is shapped like a cylinder of height 5 in with a hemisphere (a half of sphere) dome on top. The radius of this object is 3 in.*

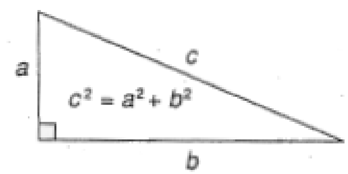


1.6. **Solving Problems Involving Right Triangles.**

Theorem. *If c is the length of the longest side (hypotenuse) of a right triangle and a and b are the lengths of the shorted sides (legs), then*

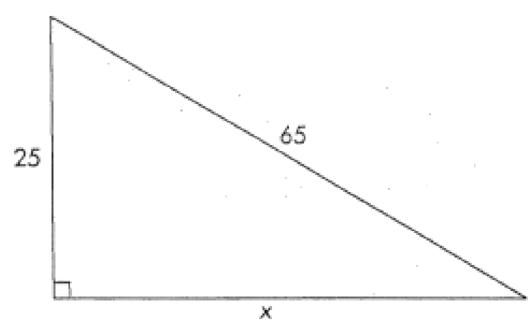
$$\boxed{c^2 = a^2 + b^2}$$

Which is called Pythagorean formula.



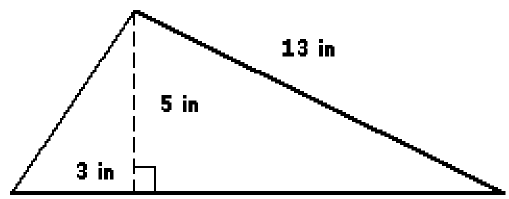
The Pythagorean theorem

Problem. *Find the length x of the unknown side of the right triangle in the drawing.*



Problem. *One end of a 50-foot wire is connected to the top of a pole. The other end of the wire is connected to a stake at a distance of 30 feet from the bottom of the pole. How tall is the pole?*

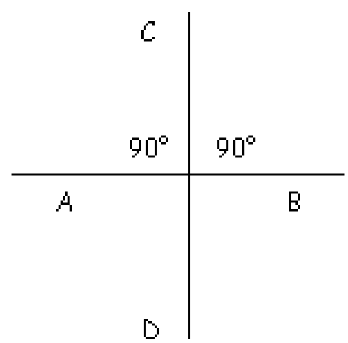
Problem. *Find the area of the figure in the drawing.*



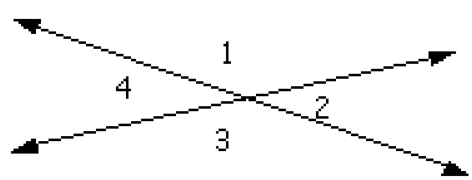
1.7. **Solving Problems with Parallel and Perpendicular Lines.**

Definition. Two angles A and B are congruent, written $\angle A \cong \angle B$ if they have the same measure.

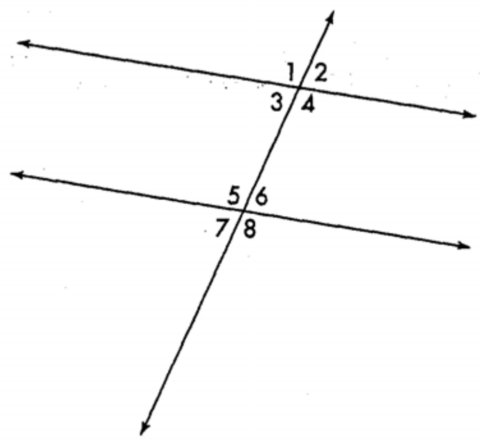
- If two lines from congruent adjacent angles, then they are perpendicular ($AB \perp CD$).



- Vertical angles are congruent. ($\angle 1 \cong \angle 3$ and $\angle 2 \cong \angle 4$).



- If two parallel lines are intersected by a transversal, then
  - Alternate interior angles are congruent $\angle 3 \cong \angle 6$ and $\angle 4 \cong \angle 5$.
  - Alternate exterior angles are congruent $\angle 1 \cong \angle 8$ and $\angle 2 \cong \angle 7$.
  - Corresponding angles are congruent $\angle 1 \cong \angle 5$, $\angle 2 \cong \angle 6$, $\angle 3 \cong \angle 7$, and $\angle 4 \cong \angle 8$.

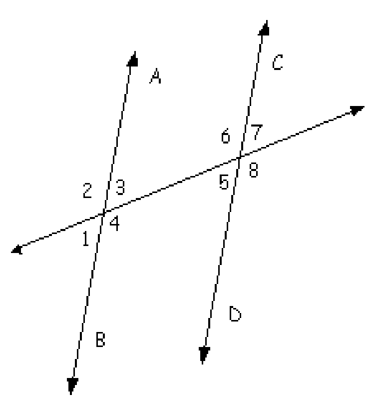


Transversal

- If two lines are intersected by a transversal so that a pair of corresponding angles are congruent, then the two lines are parallel.
- If two lines are intersected by a transversal so that two interior angles on the same side of the transversal are supplementary, then the two lines are parallel.
- If two lines are intersected by a transversal so that two interior angles on the same side of the transversal are supplementary, then the two lines are parallel.

Problem. *Use the diagram to answer the questions below.*

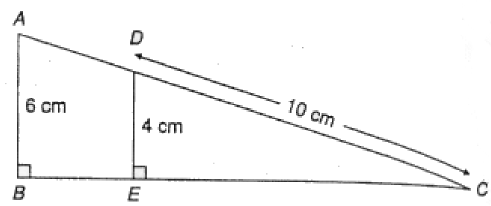
- *The measure of* $\angle 1$ *is* 50^0, *what is the measure of* $\angle 3$*?*
- *If the measure of* $\angle 8$ *is* 130^0, *what is the measure of* $\angle 7$*?*
- *If the line* AB *is parallel to line* CD, *and the measure of* $\angle 3$ *is* 50^0, *what is the measure of* $\angle 5$*?*
- *If line* AB *is parallel to line* CD, *what is the sum of the measures of* $\angle 2$ *and* $\angle 7$*?*



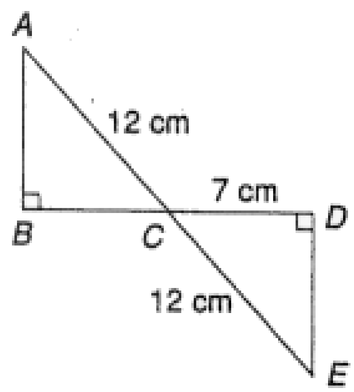
1.8. **Problems Involving Congruence and Similarity.**

Definition. Two figures are *similar* if they have the same shape.
Two figures are *congruent* if the have the same shape and the same size.

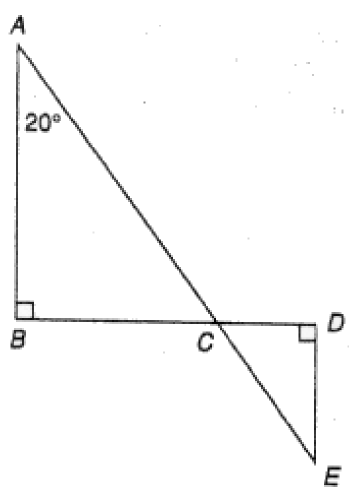
Problem. *In the diagram, what is the measure of AC?*



Problem. *In the diagram, what is the measure of BC?*



Problem. *In the diagram, what is the measure of* $\angle CED$



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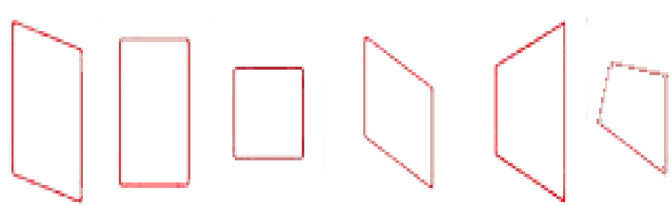
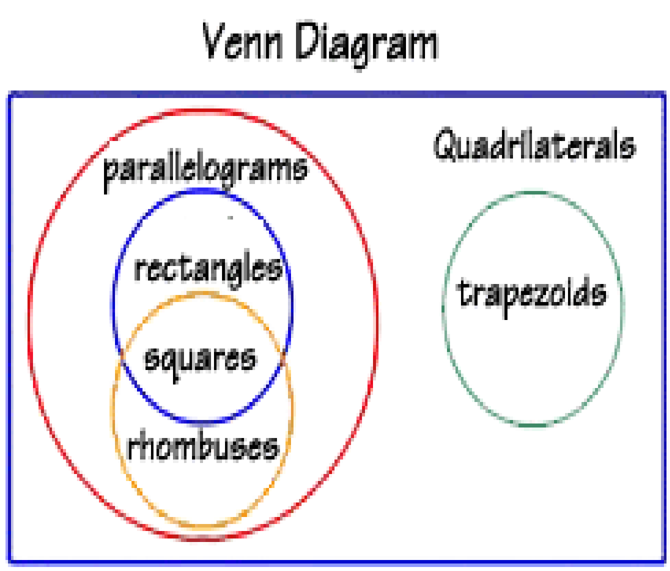
17.2 Organizing Shapes.

Definition. Quadrilaterals. A quadrilateral is a four-sided polygon with four angles. There are many kinds of quadrilaterals. The five most common types are the parallelogram, the rectangle, the square, the trapezoid, and the rhombus.

- A **parallelogram** has two parallel pairs of opposite sides.
- A **rectangle** has two pairs of opposite sides parallel, and four right angles. It is also a parallelogram, since it has two pairs of parallel sides.
- A **square** has two pairs of parallel sides, four right angles, and all four sides are equal. It is also a rectangle and a parallelogram.
- A **rhombus** is defined as a parallelogram with four equal sides. Is a rhombus always a rectangle? No, because a rhombus does not have to have 4 right angles.
- A **kite** is a quadrilateral having two different pairs of congruent adjacent sides, in contrast to a parallelogram, where the congruent sides are opposite.
- A **trapezoid** is a quadrilateral having only one pair of parallel sides. It's a type of quadrilateral that is not a parallelogram.





Using the regular format for homework: Page 408 all problems 1-6

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17.3 Triangles and Quadrilaterals.

Definition. Proposition, Theorem, Axiom, Postulate, Lemma, Corollary.

- A **proposition** p is a statement that can be true or false but not both things at the same time.
- An **open propostion** $p(x)$ is a statement that depending of the value of its variable it can true or false but not both things at the same time.
- A **theorem** is a proven proposition.
- a **proof** is a convincing demonstration (within the accepted standards of the field) that some mathematical statement is necessarily true. Proofs are obtained from deductive reasoning, rather than from inductive or empirical arguments. That is, a proof must demonstrate that a statement is true in all cases, without a single exception. An unproved proposition that is believed to be true is known as a **conjecture**.
- An **axiom** o postulate is a true proposition.
- A **lemma** is a proven theorem used as a stepping-stone toward the proof of another more important theorem.
- A **corollary** is a theorem which follows readily from a previously proven theorem.
- Any properly stated **definition** is an example of a **biconditional sentence**.
- A **counterexample** is an example that shows that a statement is not true in general.
- A **logical connector** combines simple statements into compound statements. Another name for a logical connector is logical operator: $\wedge$, $\vee$, $\neg$, $\implies$, $\iff$, etc.

Definition. Negation $\neg p$, is defined as

	Negation
p	$\neg p$
T	F
F	T

$\neg p$ is readed as: not p

Definition. Disyuntion $p \vee q$, is defined as

		Disyuntion
p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

$p \vee q$ is readed as:

a) p vel q
b) p or q

Definition. Conjuntion $p \wedge q$, is defined as

		Conjuntion
p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

$p \wedge q$ is readed as:

a) p et q
b) p and q

Definition. Implication $p \implies q$, is defined as

p	q	$p \implies q$
T	T	T
T	F	F
F	T	T
F	F	T

$p \implies q$ is readed as:

a) p seq q
b) If p, entonces q
c) p implies q

Definition. Biconditional or double implication $p \iff q$, $(p \equiv q)$ is defined as

		Biconditional
p	q	$p \iff q$
T	T	T
T	F	F
F	T	F
F	F	T

$p \iff q$ $(p \equiv q)$ is readed as:

a) p aeq q
b) p, if and only if q
c) p is equivalent to q

Problem 1. *Using the previous tables, discusse the statements* $p =$ *"I kiss you",* $q =$*I take you to the movies."*

Problem 2. *Let* p *be "Today is Friday" and let* q *be "Today is Monday".*

1) *Write the disjunction of the two statements.*
2) *Write the conjunction of the two statements.*
3) *Write the negation of statement* p*.*

Problem 3. *Let* p *be "Kevin is cooperative" and let* q *be "John is uncooperative," write each statement in symbolic form.*

1) *Kevin and John are both cooperative.*
2) *Neither Kevin nor John is uncooperative.*
3) *It is not the case that Kevin and John are both uncooperative.*
4) *Either Kevin is cooperative or John is uncooperative.*

Problem 4. *Construct a truth table for* $(p \vee q) \vee (r \wedge \neg q)$*.*

Problem 5. *Show that* $\neg(p \wedge q)$ *and* $\neg p \vee \neg q$ *are equivalent.*

Problem 6. *Find the truth value of each statement.*

1) *If* $2 + 2 = 4$*, then* $8 = 5$*.*
2) *If* $2 + 2 = 22$*, then* $8 = 4 + 4$*.*
3) *If* $2 + 2 = 22$*, then* $4 = 26$*.*

Problem 7. *Let* p *be "I kiss you once," and let* q *be "I kiss you again." Under which condition is the statement* $p \implies q$ *false?*

Problem 8. *Construct a truth table for* $[(p \implies q) \implies p] \implies q$*.*

Problem 9. *Using a truth table show that* $\neg(p \implies q)$ *is equivalent to* $p \wedge \neg q$*.*

Problem 10. *Do problem 6 page 417.*

Problem 11. *Do problem 8 page 417.*

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20.1 Size Changes and Similarity.

Problem 1. *Do activity 1 from page 447.*

Problem 2. *Do activity 2 from page 449.*

Problem 3. *Do activity 3 from page 449.*

Definition. Enlargements or miniatures must have the exact shape as the original. Two shapes related in this way are called **similar**.

$$\text{scale factor} = \frac{\text{new length}}{\text{corresponding original length}}$$

or

$$\text{scale factor} = \text{new length} : \text{corresponding original length}$$

Problem 4. *Do example 1 from page 453.*

Problem 5. *Do example 2 from page 453.*

Problem 6. *Do example 3 from page 454.*

Problem 7. *Do problem 8 from page 457.*

Problem 8. *Do problem 19 from page 459.*

Remark. Examples:

- 3.4 : 2, it means new lenght=3.4, original lenght=2, then scale factor=$\frac{3.4}{2} = 1.7$
- 1.7 times as long as $2cm$, it means $1.7 \cdot 2 = 3.1cm$
- 170% as long as $2cm$, it means 170% $of\ 2 = 1.7 \cdot 2 = 3.4cm$
- $1.4cm$ longer than $2cm$, it means $2 + 1.4 = 3.4cm$

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20.2 More About Similar Figures.

Problem 1. *Do Activity 5 page 462.*

Problem 2. *Do Example 4 page 464.*

Problem 3. *Do Example 5 page 465.*

Problem 4. *Do Problemn 4b, 4d page 466.*

Using the regular format for homework: Page 465 all problems 1-7

20.3 Size Changes in Space Figures.

Problem 1. *Do Discussion 3 page 468.*

Problem 2. *Do Discussion 4 page 468.*

Proposition. *Two 3D shapes are similar if the points in the two shapes can be matched so that*
1) *every pair of corresponding angles have the same size (measure), and*
2) *the ratios from every pair of corresponding lenght all equal the same value, called the scale factor.*

$$scale\ factor = \frac{new\ length}{corresponding\ original\ length}$$

Problem 3. *Do the activity from page 470.*

Problem 4. *Do problem 14 page 473.*

Problem 5. *Using a rectangular prism, obtain the following formulas:*

$$n.l. = (s.f.)(o.l.) \quad n.a. = (s.f.)^2(o.a.) \quad n.v. = (s.f.)^3(n.v.)$$

Problem 6. *Do problem 22 page 474.*

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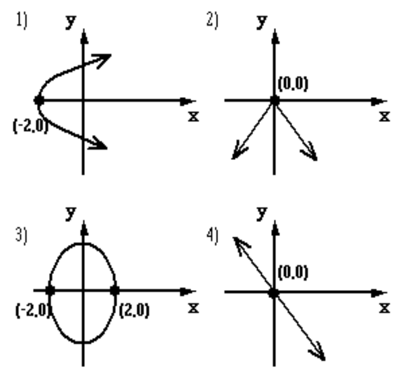
21.1 Planar Curves and Construction.

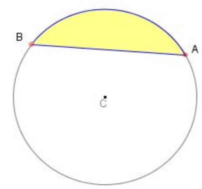
Definition. A planar curve is a curve that lie in a single flat surface. E.g.



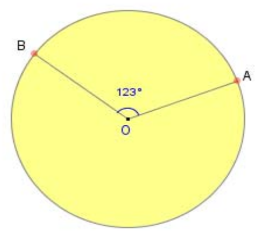
Definition. Properties of a Circle.

- The **Center** is a point inside the circle. All points on the circle are equidistant (same distance) from the center point.
- The **Radius** (plural radii) is the distance from the center to any point on the circle. It is half the diameter.
- The **Diameter** is the distance across the circle. The length of any chord passing through the center. It is twice the radius.
- The **Circumference** is the distance around the circle.
- A **Chord** is a line segment linking any two points on a circle.
- A **Tangent** is a line passing a circle and touching it at just one point.
- A **Secant** is a line that intersects a circle at two points.
- An **Arc** is a portion of the circumference of a circle. Strictly speaking, an arc could be a portion of some other curved shape, such as an ellipse, but it almost always refers to a circle. An angle with its vertex at the center of a circle cuts off (intercept) a piece of the circle, called an arc of the circle. Sometimes an arc is measured by the size of that angle, rather than by its actual length. A common notation for the arc with end points A and B is $\widehat{AB}$.
- A **Circular Sector** is the area enclosed by two radii of a circle and their intercepted arc. A pie-shaped part of a circle.

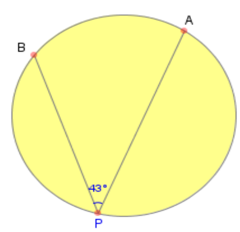
- A **Segment of a Circle** is the area of the region between a chord of a circle and its associated arc.



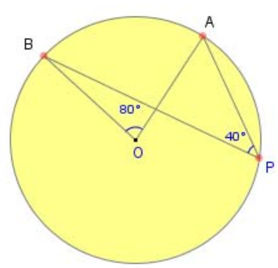
- The **Central Angle** the angle subtended at the center of a circle by two given points on the circle.



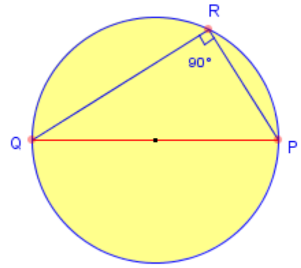
- An **Inscribed Angle** is the angle subtended at a point P on the circle by two given points on the circle. The inscribed angle is only defined for points on the major arc (the longest path around the circle between the two given points). In the figure above, if you drag P around into the shorter (minor) arc it will be undefined.



- The **Central Angle Theorem:** the central angle subtended by two points on a circle is twice the inscribed angle subtended by those points.

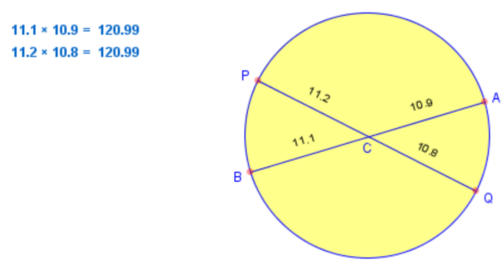


- **Thales' Theorem:** The diameter of a circle always subtends a right angle to any point on the circle.

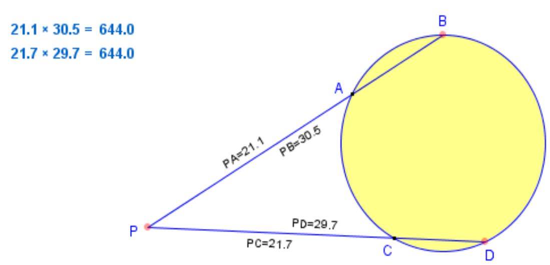


A practical application - finding the center of a circle The converse of Thales Theorem is useful when you are trying to find the center of a circle. A right angle whose vertex is on the circle always "cuts off" a diameter of the circle. That is, the points P and Q are always the ends of a diameter line. Since the diameter passes through the center, by drawing two such diameters the center is found at the point where the diameters intersect.

- **Intersecting Chord Theorem:** When two chords intersect each other inside a circle, the products of their segments are equal.



- **Intersecting Secants Theorem:** When two secant lines intersect each other outside a circle (point P), the products of their segments are equal.
(Note: Each segment is measured from the outside point P).



Problem 1. *Prove the Central Angle Theorem.*

Problem 2. *Prove the Thales' Theorem.*

Problem 3. *Prove the Intersecting Chord Theorem.*

Problem 4. *Prove the Intersecting Secants Theorem (homework).*

Problem 5. *Do Discussion 1, page 478.*

Problem 6. *Do Activity 1, page 479.*

Problem 7. *Do problem 6, page 483.*

Problem 8. *Do problem 13, page 486.*

Problem 9. *Do activities from page 480, 481, and 482.*

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23.1-23.2 Key Ideas of Measurement and Length and Angle Size.

Definition. The **process of direct measurement** of a characteristic involves matching that chararcteristic of the object by using (often repeatedly) a unit. We usually use **standard units** when giving values of quantities.
Most of the world uses the metric system, or **SI** (International System of Units).

Definition. The standard unit to measure angles is the **degree** ^{o}. One full turn is divided in 360^o. 1 degree is divided in $60'$ (minutes), and 1 minute in $60''$ (seconds). The metric system reconizes the degree as a unit, but the official SI unit is called the **radian** (about (57.3^o).

Definition. An angle of 1 radian is defined to be the angle, in the counterclock wise direction, at the center of a circle of radius r which spans an arc of length r.

$$\boxed{\theta = \frac{s}{r}}$$

where θ is measured in radians.

Proposition. π *radians*$=180°$.

Proposition. *The arc length, s, subtended in a circle of radius r by an angle of θ radians, $0 \le \theta \le 2\pi$, is given by*

$$s = r\theta$$

Problem 1. *Do Discussion 1-2 page 526.*

Problem 2. *Do Activity 1 page 529.*

Problem 3. *Do Discussion 3-5 page 530.*

Problem 4. *If the length of some object is reported as $2\frac{3}{4}$, to the nearest $1/4$. Find the interval where you will approximate any measurement to this particular value.*

Problem 5. *Do Problem 14(a) page 533.*

Problem 6. *Do Problem 16 (a) and (f) page 533.*

Problem 7. *Do Problem 18 (a) and (d) page 534.*

Problem 8. *Do Problem 20 page 534.*

Problem 9. *Do Discussion 6 page 536.*

Problem 10. *Do Problem 16 page 543.*

Problem 11. *Do Problem 18 page 543.*

Problem 12. *Do Problem 20 page 543.*

Using the regular format for homework: Page 531 all odd problems 1-19

Using the regular format for homework: Page 540 all odd problems 1-25