Instructor: Jorge R. Viramontes
MATHEMATICS 0310

R.1 Fractions.

Definition. In a fraction $\frac{a}{b}$, a is called the **numerator** and b is called the **denominator**. A fraction is **simplified** (or **reduced to its lowest terms** when the numerator and denominator have no common factors other than 1.
A fraction where $a < b$ is called **proper fraction**, a fraction where $a > b$ is called **improper fraction**. An improper fraction can be written as a mixed number.

Procedure. *Simplification of Fractions.*

Step1.- *Find the prime factorization of the numerator.*
Step2.- *Find the prime factorization of the denominator.*
Step3.- *Cancel common factors.*

Problem 1. *Simplify each fraction.*

1.- $\frac{3}{9}$

2.- $\frac{60}{105}$

3.- $\frac{80}{125}$

Problem 2. *Convert each mixed number to a improper fraction.*

1.- $1\frac{7}{8}$

2.- $2\frac{13}{15}$

3.- $3\frac{3}{32}$

Problem 3. *Write each fraction as a mixed number.*

1.- $\frac{4}{3}$

2.- $\frac{32}{7}$

Problem 4. *Find each product.*

1.- $\frac{1}{2} \cdot \frac{3}{4}$

2.- $2\frac{1}{5} \cdot \frac{7}{8}$

3.- $\frac{5}{12} \cdot \frac{4}{15}$

Problem 5. *Find each quotient.*

1.- $5\frac{3}{8} \div 1\frac{1}{4}$

2.- $4\frac{4}{5} \div \frac{8}{15}$

3.- $\frac{\frac{2}{3}}{\frac{3}{5}}$

Problem 6. *Find each addition or subtraction.*

1.- $\frac{5}{12} - \frac{1}{8}$

2.- $5\frac{3}{4} - \frac{1}{3}$

3.- $2\frac{1}{3} + 1\frac{1}{8}$

Instructor: Jorge R. Viramontes
MATHEMATICS 0310

1.1 Sets of Numbers and the Real Number System.

Definition. A **Set** is a collection of elements listed within braces. The set $\{a, b, c, d, e\}$ consists of five elements, namely a, b, c, d, and e. A set that contains no elements is called an **empty set** (or **null set**). The symbols $\{\ \}$, or ϕ are used to represent the empty set.

Definition. The following number sets are important:

- Natural Numbers: $\mathbb{N} = \{1, 2, 3, \ldots\}$

- Whole Numbers: $\mathbb{W} = \{0, 1, 2, 3, \ldots\}$

- Integers $\mathbb{Z} = \{\ldots, -3, -2, -1, 0,\ 1, 2,\ 3, \ldots\}$
  The numbers -3, -2, -1 are the opposite or negative of 3, 2, 1.

- Rational Numbers:
$$\begin{aligned}\mathbb{Q} &= \{quotient\ of\ two\ integers,\ denominator\ not\ 0\}\\ &= \{\frac{a}{b} :\ \ a, b \in \mathbb{Z} \text{ and } b \neq 0\}\end{aligned}$$
  $\frac{a}{b}$ is a quotient of integers. e.g. $\frac{1}{5}$, $\frac{4}{3}$, 7 (since $7 = \frac{7}{1}$)

- Irrational Numbers: $\mathbb{I} = \{$All the numbers that can not be written as a quotient of integers$\}$
  e.g. $\sqrt{2}$, $\sqrt{3}$, π, ...

- all these numbers can be represented on a number line called the **Real number line**

  Real numbers:
$$\begin{aligned}\mathbb{R} &= \{Irrational\ and\ Rational\ numbers\}\\ &= \{all\ the\ numbers\ that\ can\ be\ represented\ on\ a\ real\ number\ line\}\end{aligned}$$

Definition. The Ordering of Real Numbers.

a) For any two real numbers a and b, a is less than b ($a < b$) if a is to the left of b on the number line.

b) For any two real numbers a and b, b is greater than a ($b > a$), if $a < b$.

Problem 1. *Tell whether each statement is true or false (use the number line).*

1.- $-2 < 4$

2.- $6 > -3$

3.- $-9 < -12$

4.- $-4 \geq -1$

Problem 2. *Use either $>$, $<$, or $=$ in each of the following.*

1.- $\frac{2}{3} \cdot \frac{2}{3}$ __ $\frac{2}{3} + \frac{2}{3}$

2.- $5 \div \frac{2}{3}$ __ $\frac{2}{3} \div 5$

3.- $\frac{5}{8} - \frac{1}{2}$ __ $\frac{5}{8} \div \frac{1}{2}$

4.- $2\frac{1}{3} \cdot \frac{1}{2}$ __ $2\frac{1}{3} + \frac{1}{2}$

Definition. Two numbers that are the same distance from 0 but on opposite sides of 0 on the number line are called **opposite** of each other. Symbolically, we denote the opposite of a real number a as $-a$.

Problem 3. *Finding the opposite of a real Number.*

a) *Find the opposite of* 5.

b) *Find the opposite of* $-\frac{4}{7}$.

c) *Evaluate* $-(0.46)$.

d) *Evaluate* $-\left(-\frac{11}{3}\right)$

Definition. The **absolute value** of a number a, written $|a|$, can be considered the distance between the number a and 0 on a number line. E.g.
The absolute value of 3 written $|3|$, is 3 since it is 3 units from 0 on a number line. Similarly the absolute value of -3 written $|-3|$ is also 3 since -3 is 3 units from 0.

Problem 4. *Use either $<$, $>$, or $=$ in each of the following.*

1.- $\left|-\frac{6}{2}\right|$ __ $\left|-\frac{2}{6}\right|$

2.- 4 __ $\left|-\frac{9}{2}\right|$

3.- $|-4|$ __ -3

Problem 5. *Calculate the following.*

1.- $|4-9| =$

2.- $-|4-9| =$

Problem 6. *Find an expression for* $|x|$.

Problem 7. *Give three real numbers that satisfy all the stated criteria.*

1.- *greater than* 4 *and less than* 6.

2.- *less than* -2 *and greater than* -6.

3.- *greater than* $|-2|$ *and less than* $|-6|$.

4.- *greater than* $|-3|$ *and less than* $|3|$.

Instructor: Jorge R. Viramontes
MATHEMATICS 0310

1.2 Order of Operations.

Definition. A **variable** is a symbol or letter such as x, y, and z, used to represent an unknown number. **Constants** are values that are not variable such as the numbers 3, -1.5, $\frac{2}{7}$, and π. An algebraic **expression** is a collection of variables and constants using algebraic operations. For example $\frac{3}{x}$, $y+7$, and $t-1.4$ are algebraic expressions.
The symbols used to show the four basic operations of addition, subtraction, multiplication, and division are:

Operation	Symbol	Translation
Addition	$a+b$	sum of a and b; a plus b; b added to a; b more than a; a incresead by b; the total of a and b
Subtraction	$a-b$	difference of a and b; b subtracted from a; a decreased by b; b less than a
Multiplication	$a$xb, $a\cdot b$, $a(b)$, $(a)(b)$, $(a)b$	product of a and b; a times b; a multiplied by b
Division	$a \div b$, $\frac{a}{b}$, a/b	quotient of a and b; a divided by b; b divided into a; ratio of a and b; a over b; a per b.

Definition. The expression a^n is called exponential expression, the number a is the base, and the number n is the exponent:

$$a^n = \overbrace{a\cdot a\cdot a\cdot a\cdots a}^{n-times}$$

Problem 1. *Find the value of each exponential expression.*

a.- -6^2

b.- $(-6)^2$

c.- $\left(\frac{3}{4}\right)^2$

d.- $\left(\frac{1}{2}\right)^4$

e.- $(.4)^3$

Procedure. *Order of Operations.*
Step 1 *If possible simplify parentheses from inside to outside and above or below fractions bars.*
Step 2 *Apply all exponents.*
Step 3 *Do Multiplications or Divisions in the order in which they occur, working from left to right.*
Step 4 *Do Additions or Substractions in the order in which they occur, working from left to right.*

Problem 2. *Find the value of each expression.*

a.- $6 - |-2| + \sqrt{15 - 6}$.

b.- $5 + 3 \cdot [1 + (2 \cdot 3)] + 1$.

c.- $\dfrac{5 - [3(6 \div 3) - 2]}{5^2 - 4^2 \div 2}$.

d.- $3^2 \cdot 2^2 + 2^2 + 3^2$.

Problem 3. *Find the value of each expression if $x = 6$ and $y = 9$*

a.- $4x + 7y$

b.- $\dfrac{4x - 2y}{x + 1}$

c.- $2x^2 + y^2$

Problem 4. *Translate each English phrase into an algebraic expression.*

a) *The quotient of x and 5*
b) *Seven less than n*
c) *Eight more than the absolute value of w*
d) *The product of a and the square root of b*
e) *The difference of twice a and b*

Instructor: Jorge R. Viramontes
MATHEMATICS 0310

1.3 1.4, 1.5 Addition , Subtraction, Multiplication, and Division of Real Numbers.

Definition. Substraction. For any real numbers a and b,

$$a - b = a + (-b)$$

a minus b means a plus the opposite of b.

Procedure. *Operations with Signed Numbers.*

- *Addition/Subtraction*
  like signs *Add the absolute values of the numbers. The result is given the same sign as the numbers,*
  $$(+) + (+) = +$$
  $$(-) + (-) = -$$
  e.g.
  $$+5 + 2 = +7$$
  $$-5 - 2 = -7$$
  (we omit the + sign at the beginning of an expression, before a - sign, or after the = sign so, $+5 + 2 = +7$ *is the same as* $5 + 2 = 7$*)*
  Unlike signs *Subtract them (largest absolute value number - smallest absolute value number) and then give the result the sign of the number having the largest absolute value, e.g.*
  $$-2 + 5 =$$
  Subtract them (5-2=3 largest absolute value number - smallest absolute value number) since 5 *has the largest absolute value between* 2 *&* 5*, then* $-2 + 5 = +3$
  $$2 - 5 =$$
  Subtract them (5-2=3 largest absolute value number - smallest absolute value number) since 5 *has the largest absolute value between* 2 *&* 5*, then* $2 - 5 = -3$

- *Multiplication/Division*
  like signs *The product or quotient of two numbers with like signs is positive.*
  $$(+)(+) = + \quad or \quad (+)/(+) = +$$
  $$(-)(-) = + \quad or \quad (-)/(-) = +$$
  $$(5)(2) = +10 \quad 15/3 = 5$$
  $$(-5)(-2) = +10 \quad -15/-3 = 5$$
  unlike signs *The product or quotient of two numbers with unlike signs is negative.*
  $$(+)(-) = - \quad or \quad (+)/(-) = -$$
  $$(-)(+) = - \quad or \quad (-)/(+) = -$$
  $$(+5)(-2) = -10 \quad 15/-3 = -5$$
  $$(-5)(+2) = -10 \quad -15/3 = -5$$

Definition. For any number a,

- $a \cdot 0 = 0$
- $\frac{a}{b} = n$, if and only if $n \cdot b = a$.

Problem 1. *Calculate the following.*

1.- $\frac{3}{12} \div (-\frac{5}{8})$

2.- $\left(-\frac{3}{8}\right)\left(\frac{5}{6}\right)$

3.- $-\frac{7}{12} \div (-2)$

4.- *Suppose that a represent a positive number and b represent a negative number. Find whether the given expressions are positive or negative:*

$$a - b \qquad b - a \qquad a + |b| \qquad b - |a|$$

Problem 2. *If a represents any real number except 0, find $\frac{0}{a}$, $\frac{a}{0}$.*

Proposition. *Let a be positive real number, then the following operations are true.*

$$\begin{aligned} +(+a) &= +1(+a) = a \\ +(-a) &= +1(-a) = -a \\ -(+a) &= -1(+a) = -a \\ -(-a) &= -1(-a) = -a \end{aligned}$$

Problem 3. *Simplify.*

1) $+3 - (-2) + =$

2) $-(+2) + (-3) =$

Instructor: Jorge R. Viramontes
MATHEMATICS 0310

1.6 Properties of the Real Number System.

Properties of Addition and Multiplication:
$\forall$ means "for every", $\exists$ means "there exists", $\in$ "belongs to", $\ni$ "such that", $\mathbb{R}$ Real numbers set.

- Commutative Property:

$$\forall x,\ y \in \mathbb{R}$$
$$x + y = y + x$$
$$x \cdot y = y \cdot x$$

- Associative Property:

$$\forall x,\ y,\ z \in \mathbb{R}$$
$$x + (y + z) = (x + y) + z$$
$$x \cdot (y \cdot z) = (x \cdot y) \cdot z$$

- Identity Property:

$$\exists 0 \in \mathbb{R} \ni \forall x \in \mathbb{R}$$
$$x + 0 = x$$
$$\exists 1 \in \mathbb{R} \ni 1 \neq 0 \text{ and } \forall x \in \mathbb{R}$$
$$x \cdot 1 = x$$

0 is called the identity element of addition, and 1 is called the identity element of multiplication.

- Inverse Property:

$$\forall x \in \mathbb{R} \exists -x \in \mathbb{R} \ni$$
$$x + (-x) = 0$$
$$\forall x \in \mathbb{R}/\{0\}\ \exists \frac{1}{x} \in \mathbb{R} \ni$$
$$x \cdot \frac{1}{x} = 1$$

$-x$ is called the opposite of x or the additive inverse element, and $\frac{1}{x}$ the reciprocal of x or the multiplicative inverse element.

- Distributive Property:

$$\forall x,\ y,\ z \in \mathbb{R}$$
$$x \cdot (y + z) = x \cdot y + x \cdot z$$

Problem 1. *Complete each statement. Use a commutative property.*

1.- $x + 9 =$

2.- $(-12)(4) =$

3.- $5x =$

Problem 2. *Complete each statement. Use an associative property.*

1.- $(9 + 10) + (-3) =$

2.- $-5 + (2 + 8) =$

3.- $10 \cdot [(-8) \cdot (-3)] =$

Problem 3. *Decide whether each statement is an example of a commutative property, an associative property, or both.*

1.- $2(4 \cdot 6) = (2 \cdot 4)6$

2.- $(2 \cdot 4)6 = (4 \cdot 2)6$

3.- $(2 + 4) + 6 = 4 + (2 + 6)$

4.- *Simplify* $8 + 4y + 10$

Problem 4. *Use an identity property to complete each statement.*

1.- $9 + 0 =$ ____

2.- ____ $+ (-7) = -7$

3.- $\frac{1}{4} \cdot$ ____ $= \frac{3}{12}$

4.- ____ $\cdot 1 = 5$

Problem 5. *Complete the statement so that they are examples of either an identity property or an inverse property. Tell which property is used.*

1.- $-6 +$ ____ $= 0$

2.- $\frac{4}{3} \cdot$ ____ $= 1$

3.- $-\frac{1}{9} \cdot$ ____ $= 1$

4.- $275 +$ ____ $= 275$

5.- *Simplify* $5m - 3 - 5m$

Problem 6. *Use distributive property to rewrite each expression.*

1.- $2(p + 5)$

2.- $-4(y + 7)$

3.- $5(m-4)$

4.- $9 \cdot k + 9 \cdot 5$

5.- $3a - 3b$

6.- $7(2y + 7k - 9m)$

7.- $-(3k - 5)$

8.- $-(2 - r)$

9.- *Simplify* $4x + x$

Problem 7. *Fill in the blanks with the correct responses.*

1.- *The identity element for addition is* ____________________.

2.- *The identity element for multiplication is* ____________________.

3.- *Every number has a(n)* ____________________ *inverse.*

4.- *Every number except* ____________________ *has a(n)* ____________________ *inverse.*

5.- *The sum of a number and its* ____________________ *is 0.*

6.- *The* ____________________ *of a number and its* ____________________ *is 1.*

7.- *The additive inverse of a is* ____________________.

8.- *The multiplicative inverse of a* $(a \neq 0)$ *is* ____________________.

Instructor: Jorge R. Viramontes
MATHEMATICS 0310

Review: Simplifying Expressions.

Definition. -

- A term is an expression separated by the + or − sign.
- Terms with exactly the same variables including the same exponents are called like terms.
- A factor is an expression separated by the $\cdot$ sign.
- The numerical factor of a term is the called the coefficient of the term.
- An algebraic expression with more than one term is called *polynomial*.
- A polynomial with exactly three terms is called *trinomial*, a polynomial with exactly two terms is called a *binomial*, an algebraic expression with exactly one term is called a *monomial*.

Problem. *Simplify each expression.*

1.- $2 - 3x - 2x + y$

2.- $2y^2 + 4 - x^2 + 5x + 3y^2 - 5x^2 + 8$

3.- $\frac{3}{5}x - 3 - \frac{7}{4}x - 2$

4.- $\frac{1}{2}y - 4 + \frac{3}{4}x - \frac{1}{5}y$

Problem. *Use the distributive property to remove parentheses.*

1.- $-2(x + y - z)$

2.- $2(3x - 2y + \frac{1}{4})$

3.- $\frac{1}{2}(-2x + 6)$

4.- $-3(-x + 4 + 2y)$

5.- $-\frac{1}{2}(2x - 4)$

Problem. *Simplify.*

1.- $4 + (3x - 4) - 5$

2.- $4 - (y - 5) - 2x + 1$

3.- $-6x + 7y - (3 + x) + (x + 3)$

4.- $4(x - 1) + 2(3 - x) - 4$

5.- $-5(2y - 8) - 3(1 + x) - 7$

Instructor: Jorge R. Viramontes
MATHEMATICS 0310

2.1 The Addition & Multiplication Properties of Equality.

Definition. .

- A statement that shows two algebraic expressions are equal is called an **equation**.
- An equation having variables with exponent equal to 1, is called a **linear equation**.
- A linear equation written in the form $ax + by = c$ is called a linear equation in its **standard form**, where a, b, and c are real numbers, with $a \neq 0$.
- Solving an equation means find the value of the variable(s) which makes the equation a true statement.
  To solve an equation, we add or multiply the same number to each side of this equation. The addition and multiplication properties justify this step.
- Equations that have exactly the same solutions are **equivalent equations**.

Proposition. Addition Property of Equality. *If a, b, and c are real numbers, then the equations*

$$a = b \quad \text{and} \quad c + a \ = \ b + c$$

are equivalent equations. That is, we can add to each side of an equation the same number without changing the equation solution.

Proposition. Multiplication Property of Equality. *If a, b, and c ($c \neq 0$) are real numbers, then the equations*

$$a = b \quad \text{and} \quad c \cdot a \ = \ b \cdot c$$

are equivalent equations. That is, we can multiply each side of an equation by the same nonzero number without changing the equation solution.

Problem 1. *Answer the questions.*

1.- *Is* $x = \frac{1}{2}$ *a solution of* $x + 3 = 3x + 2$*?*

2.- *Is* $x = 3$ *a solution of* $3x - 3 = 9$*?*

Problem 2. *Solve each equation and check your solution.*

1.- $x - 4 = 13$

2.- $5 + x = 18$

3.- $\frac{9}{2}m + 1 = \frac{7}{2}m$

4.- $-(5 - 4r) + 3(-r + 1) = 1$

5.- $-3(m - 4) + 2(5 + 2m) = 29$

6.- $2(x - 6) = 2x - 12$ *Special Case*

7.- $3x + 6(x + 1) = 9x - 4$ *Special Case*

Problem 3. *Solve (apply the multiplication property).*

1.- $3x = 15$ *and check by substituting back in the given equation.*

2.- $-6p = -14$

3.- $3r = -12$

4.- $-2m = 16$

5.- $\frac{y}{5} = 5$

6.- $\frac{p}{4} = -6$

7.- $-\frac{5}{6}t = -15$

Instructor: Jorge R. Viramontes
MATHEMATICS 0310

2.2 Solving linear equations.

(variable on only one side of the equation)

Problem 1. *Solve each equation.*

1.- $2(3x-4)-4x=12$

2.- $3(4-x)+5x=9$

3.- $1+(x+3)+6x=6$

4.- $5x+3x-4x-7=9$

5.- $3-2(x+3)+2=1$

(with the variable on both sides of the equation).

Problem 2. *Solve each equation.*

1.- $5y-7y+6y-9=3+2y$

2.- $7m-(2m-9)=39$

3.- $2(4+3r)=3(r+1)+11$

4.- $2-3(2+6z)=4(z+1)+18$

5.- $\frac{1}{4}x-4=\frac{3}{2}x+\frac{3}{4}x$

Instructor: Jorge R. Viramontes
MATHEMATICS 0310

2.3 Linear Equations: Clearing Fractions and Decimals.

Problem 1. *Solve each equation.*

1) $\frac{1}{5}x - \frac{3}{4} = \frac{1}{3}$

2) $0.05x + 0.25 = 0.2$

3) $\frac{1}{6}x - \frac{2}{3} = \frac{1}{5}x - 1$

4) $\frac{x-2}{5} - \frac{x-4}{2} = 2 + \frac{x+4}{10}$

5) $2.5x + 3 = 1.7x - 6.6$

6) $0.20(x+4) - 0.45(x+9) = 12$

2.4 Application of Linear Equations: Introduction to Problem Solving.

Procedure. *Solving word problems.*
Step1.- *Identify the main unknow quantity or variable.*
Step2.- *Identify any other unknow quantities.*
Step3.- *Translate the problem into an equation.*
Step4.- *Solve the equation.*
Step5.- *Answer the question of the problem.*

Definition. The symbols used to show the four basic operations of addition, subtraction, multiplication, and division are:

Operation	Symbol	Translation
Addition	$a+b$	sum of a and b; a plus b; b added to a; b more than a; a incresead by b; the total of a and b
Subtraction	$a-b$	difference of a and b; b subtracted from a; a decreased by b; b less than a
Multiplication	$a$xb, $a \cdot b$, $a(b)$, $(a)(b)$, $(a)b$	product of a and b; a times b; a multiplied by b
Division	$a \div b$, $\frac{a}{b}$, a/b	quotient of a and b; a divided by b; b divided into a; ratio of a and b; a over b; a per b.

Problem 1. *The sum of a number and negative eleven is negative fifteen. Find the number.*

Problem 2. *Forty less than five times a number is fifty-two less than the number. Find the number.*

Problem 3. *Twice the sum of a number and six is two more than three times the number. Find the number.*

Problem 4. *The sum of two consecutive odd integers is* -188. *Find the integers.*

Problem 5. *Ten times the smallest of three consecutive integers is twenty-two more than three times the sum of the intergers. Find the integers.*

Instructor: Jorge R. Viramontes
MATHEMATICS 0310

2.8 Linear Inequalities.

Definition. Two algebraic expressions related by the signs $<$, $\leq$, $>$ o $\geq$ are called inequalities. E.g.

$$2x+3 \leq 4 \quad 4x > 3x-5 \quad \text{or} \quad 1.5 \leq 2.3x+4.5$$

Problem 1. *Graph each of the inequalities and use interval notation.*

a) $x < 3$

b) $x \leq 3$

c) $x > 3$

d) $x \geq 3$

e) $-2 < x < 3$

f) $-2 \leq x \leq 3$

Proposition. *Addition Property of Inequality.*
For any real numbers a, b and c, the inequalities

$$a < b \quad \text{and} \quad a+c < b+c$$

have exactly the same solution. In other words, the same number may be added to each side of an inequality without changing the solutions.

Proposition. *Multiplication Property of Inequality.*
For any real numbers a, b and c ($c \neq 0$),

- *If c is positive, then the inequalities*

$$a < b \quad \text{and} \quad ac < bc$$

  have exactly the same solution. In other words, each side of an inequality the same positive number may be multiplied by the same positive number without changing the solutions.
- *If c is negative, then the inequalities*

$$a < b \quad \text{and} \quad ac > bc$$

  If the number is negative, we must reverse the direction of the inequality symbol.

Problem 2. *Solve the following inequalities, graph each solution.*

a) $x+5(x+1) > 4(2-x)+x$

b) $8-6t \geq 2t+24$

c) $-4r + 3(r + 1) < 2r$

d) $-6 \leq 2z + 4 \leq 12$

e) $\dfrac{3(x-2)}{5} > \dfrac{5(2-x)}{3}$

Instructor: Jorge R. Viramontes
MATHEMATICS 3010

3.1 The Rectangular Coordinate System.

Definition. .

- A linear equation in two variables is an equation that can be put in the form

  $$ax + by = c \qquad \text{(the standard form of a linear equation in two variables)}$$

  where a, b, and c are real numbers and a and b are not both 0.
  A solution of a linear equation in two variables requires two numbers, one for each variable. A pair of numbers such as (x, y) is called an **ordered pair**. (pair because the two numbers, and ordered, because in general case is not the same (x, y) and (y, x).

- A graph of an equation is an illustration of a set of points whose coordinates satisfy the equation.

Problem 1. .

a) *Write the following solution as an ordered pair:* $x = 5$ *and* $y = 7$

b) *Decide whether the ordered pairs* $(0, 10)$, $(2, -5)$, *satisfy the equation:* $5x + 2y = 20$

c) *Complete the given ordered pairs for the given equation:* $y = 2x - 9$

$(5, \quad)$

$(\quad , 2)$

d) *Complete the table of ordered pairs for the equation:* $2x - 3y = 12$

x	*0*	
y		*0*

e) *Plot the points* $(4, 5)$, $(-1, 3)$, $(3, -1)$, $(\pi, 1.1)$.

Instructor: Jorge R. Viramontes
MATHEMATICS 3010

3.2-3.3 Linear Equations in Two Variables, x and y intercepts, Horizontal and Vertical Lines.

There are infinitely many ordered pairs that satisfy an equation in two variables.
For example, we find ordered pairs that are solutions of the equation $x + 2y = 7$ by choosing as many values of x or y as we wish and then completing each ordered pair.
For example, if we choose $x = 1$, then we find that $y = 3$, so that the ordered pair $(1, 3)$ is a solution of the equation $x + 2y = 7$.

Definition. -

- The intercept point between the graph of a linear equation and the $x - axis$ (i.e. when $y = 0$) is called the x-intercept of the graph.

- The intercept point between the graph of a linear equation and the $y - axis$ (i.e. when $x = 0$) is called the y-intercept of the graph.

Procedure. *Graphing a Linear Equation Finding Intercepts.*

1) *Find the x-intercept by letting $y = 0$ in the given equation and solving for x. Then $(x, 0)$ is the x-intercept.*

2) *Find the y-intercept by letting $x = 0$ in the given equation and solving for y. Then $(0, y)$ is the y-intercept.*

Proposition. .

- *If a and b are real numbers, the graph of a linear equation of the form*

$$ax + by = 0$$

  goes through the origin $(0, 0)$.

- *The graph of the linear equation $y = k$, where k is a real number, is the horizontal line going through the point $(0, k)$.*

- *The graph of the linear equation $x = k$, where k is a real number, is the vertical line going through the point $(k, 0)$.*

Problem 1. *Find the intercepts and graph.*

a) $5x + 2y = 10$

b) $3x + y = 0$

c) $2x = y$

d) $y = -5$

e) $x = 2$

Problem 2. *The cost C, of playing tennis in the Casa Blanca Tennis Club includes an annual \$200 membership fee plus \$10 per hour, h, of court time.*

- *Write an equation for the annual cost of playing tennis in terms of hours played.*
- *Graph the equation for up to and including 300 hours.*
- *Estimate the cost for playing 200 hours in a year.*
- *If the annual cost for playing tennis was \$1200, estimate how many hours of tennis were played.*

Instructor: Jorge R. Viramontes
MATHEMATICS 3010

3.4 Slope of a Line.

Definition. Slope Formula. The **slope** of the line through the points (x_1, y_1) and (x_2, y_2) is defined as

$$\boxed{m = \frac{\text{change in } y}{\text{change in } x} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}}$$

Proposition. Positive, Negative, Zero and Undefined Slopes.

- *A line with positive slope rises from left to right.*
- *A line with negative slope falls from left to right.*
- *A horizontal line ($y = k$) has a slope equal to zero.*
- *A vertical line ($x = k$) has a undefined slope.*

Problem 1. *Find the slope of each of the following lines.*

a) *Through* $(6, -2)$ *&* $(5, 4)$

b) *With equation* $3x + 2y = 6$

c) *With equation* $y = -1$

d) *With equation* $x - 4 = 0$

Proposition. *Slope of Parallel and Perpendicular Lines.*

- *Two lines such that* $m_1 = m_2$ *are parallels*
- *Two lines such that* $m_1 \cdot m_2 = -1$ *are perpendicular*

Problem 2. *Write parallel, perpendicular or neither for each pair of lines.*

1)

$$x + y = 6$$
$$x + y = 1$$

2)

$$3x - y = 4$$
$$x + 3y = 9$$

Problem 3. *Mario earns \$10.00/hr working for a landscape company. Oscar earns \$15.00/hr working for an in-hour nursing agency.*

a) *Find the slope of the line representing Mario's earnings.*

b) *Find the slope of the line representing Oscar's earnings.*

Instructor: Jorge R. Viramontes
MATHEMATICS 3010

3.5 Equations of a Line: Slope-Intercept Form

Definition. Slope-Intercept form.
The slope-intercept form of the equation of a line with slope m and y-intercept $(0, b)$ is

$$\boxed{y = mx + b}$$

Procedure. *Finding the slope and the y-intercept of a Line from its Equation.*

Solve the equation for y
The slope is given by the coefficient of x
The y-intercept is the independent term, b, that is $(0, b)$

Problem 1. *Find the slope and y-intercept of each line and graph the line.*

1) $y = -\frac{7}{2}x + 1$

2) $3x + 2y = 9$

3) $x + 3 = 7$

Problem 2. *Find the equation (in the form $ax + by = c$) of the line with the given slope and value of b.*

1) $m = -1$; $b = 8$

2) $m = 3$; $b = 0$

3) $m = 0$; $(0, 2)$

4) *Through* $(-1, 3)$, *with slope* -2

5) *Through the points* $(-3, 1)$ *and* $(2, 4)$

Summary of Linear Equations:

$ax + by = c$	**Standard Form**
$x = k$	**Vertical line** Slope is undefined x-intercept is $(k, 0)$
$y = k$	**Horizontal line** Slope is zero y-intercept is $(0, k)$
$y = mx + b$	**Slope-intercept form** Slope is m y-intercept is $(0, b)$
$m = \dfrac{y_2 - y_1}{x_2 - x_1}$	**Slope equation** Line goes through (x_1, y_1) & (x_2, y_2)

Instructor: Jorge R. Viramontes
MATHEMATICS 3010

4.1 Exponents: Multiplying and Dividing Common Bases.

Definition. In the expression

$$a^n = \overbrace{a \cdot a \cdot a \cdot a \cdots a}^{n-times}$$

a is called the base, and n is called the exponent or power. The above expression means "the product of a by itself n-times".

Proposition. *Rules for Exponents. For any integers m and n:*

1) $a^m \cdot a^n = a^{m+n}$ *-Product Rule*

2) $a^0 = 1$ *($a \neq 0$) -Zero Expoenent*

3) $a^{-n} = \dfrac{1}{a^n}$ *($a \neq 0$) -Negative Exponent*

4) $\dfrac{a^m}{a^n} = a^{m-n}$ *($a \neq 0$) -Quotient Rule*

5) *Power Rules*

   a) $(a^m)^n = a^{mn}$

   b) $(ab)^m = a^m b^m$

   c) $(\dfrac{a}{b})^m = \dfrac{a^m}{b^m}$ *($b \neq 0$)*

6) $\dfrac{a^{-m}}{b^{-n}} = \dfrac{b^n}{a^m}$ *-Negative to positive exponent*

Problem 1. *Evaluate/Simplify each exponential expression.*

1) $(-2)^5$

2) -2^5

3) -4^2

4) $(-4)^2$

5) $y^3 \cdot y^2$

6) $\dfrac{x^5}{x^2}$

7) $\dfrac{x^3}{x^5}$

8) $-(x^0)$

9) $\left(\dfrac{2}{3}m^{13}n^8\right)(24m^7n^2)$

10) $\left(\dfrac{1}{4}c^6d^6\right)(28c^2d^7)$

11) $\dfrac{w^{12}w^2}{w^4w^5}$

12) $\dfrac{z^3z^{11}}{z^4z^6}$

13) $\dfrac{t^{3+2m}}{t^{2m}}$

Instructor: Jorge R. Viramontes
MATHEMATICS 3010

4.2 More Properties of Exponents.

Problem 1. *Simplify each exponential expression.*

1) $(-5xyz)^2$

2) $\left(\frac{w}{z}\right)^7$

3) $\left(\frac{2y^3}{x}\right)^4$

4) $\left(\frac{2x^7y^2}{4xy}\right)^3$

5) $(2c^3d^2)\left(\frac{c^6d^8}{4c^2d}\right)^2$

6) $\frac{(21x^5y)(2x^8y^4)}{14xy}$

7) $(3x^4z^{10})^2(2x^2z^8)$

Instructor: Jorge R. Viramontes
MATHEMATICS 3010

4.3 Definition of b^0 and b^{-n}.

Problem 1. *Simplify each exponential expression.*

1) $(8x)^0$

2) ab^0

3) 4^{-3}

4) $\frac{4^2}{4^5}$

5) $4^{-1} - 3^{-1}$

6) $\frac{x^{-2}}{x^5}$

7) $(2x^{-3}y^{-2})(x^4y^0)$

8) $\frac{32x^4y^{-2}}{4x^{-2}y^0}$

9) $\frac{21x^{-3}z^2}{7xz^{-3}}$

10) $\frac{x^{-4}}{x^{-1}}$

11) $\frac{x^{-3}}{y^{-2}}$

12) $\frac{4h^{-5}}{m^{-2}k}$

13) $\frac{8^4 \cdot m^9}{8^5 \cdot m^{10}}$

14) $\frac{3^9 \cdot (x^2y)^{-2}}{3^3 \cdot x^{-4}y}$

Instructor: Jorge R. Viramontes
MATHEMATICS 0310

4.5 Addition and Subtraction of Polynomials

Definition. -

- **Terms** are algebraic expressions separated by the plus or minus sign.
- The numerical factor in a term is called the numerical **coefficient**.
- The **degree of a term with one variable** is the exponent on the variable.
  For example, $3x^4$ has degree 4, $6x^{17}$ has degree 17, $5x$ has degree 1, and -7 has degree 0 (since -7 can be written as $-7x^0$).
- An algebraic expression with more than one term is called **polynomial**.
- A **polynomial in** x is a polynomial with terms of the form ax^n where a is any real number and n is a whole number.
  For example $16x^8 - 7x^6 + 5x^4 - 3x^2 + 4$ is a polynomial in x.
- A polynomial is written in **descending powers of the variable or just descending order**, if the exponents on x decrease from left to right.
  On the other hand,
  $$2x^3 - x^2 + \frac{4}{x}, \quad 5\sqrt{x} - 2x, \quad \text{and} \quad x^{-2} + 4 - x^2$$
  are not polynomials in x, on the these expression n is not a whole number.
- A polynomial in x is **complete** if written the polynomial is descending order, the exponents on x decrease always by 1.
- The **degree of a polynomial** is the highest degree in any nonzero term of the polynomial. For example $3x^4 - 5x^2 + 6$ has degree 4.
- A polynomial with exactly three terms is called **trinomial**, a polynomial with exactly two terms is called a **binomial**, an algebraic expression with exactly one term is called a **monomial**.

Problem 1. *Classifying Polynomials.*

a) $3x^3 + 2x^2 - 4x - 5$

b) $9x^2 - 5x + 2x^3$

c) $3x^2 + 2x + 6 - 5x^{-1}$

Problem 2. *Find each sum.*

1) $(2x^4 - 6x^2 + 7) + (-3x^4 + 5x^2 + 2)$

2)
$$\begin{array}{l} 4x^3 - 3x^2 + 2x \\ 6x^3 + 2x^2 - 3x \end{array}$$

3) $(\frac{7}{2}y^2 - \frac{11}{3}y + 8) - (-\frac{3}{2}y^2 + \frac{4}{3}y + 6)$

Instructor: Jorge R. Viramontes
MATHEMATICS 0310

4.6 Multiplication of Polynomials.

Problem 1. -

- *Multiplication of Monomials.*

  1) $(3x^4)(4x^2)$.

  2) $\left(\frac{1}{3}a^4b^3\right)\left(\frac{3}{4}b^7\right)$.

- *Multiplying a Monomial and a Polynomial.*

  1) $5m^2(2m+7)$

  2) $-4y^2(3x^3+2y^2-4y+8)$

- *Multiplying two binomials.*

  1) $(4x+3)(2x+1)$

  2) $(3k-2)(2k+1)$

- *Multiplying any two polynomials.*

  1) $(m^3-2m+1)(2m^2+4m+3)$

  2) $(6p^2+2p-4)(3p^2-5)$

- *Multiplying Polynomials Vertically.*

  1)
$$\begin{array}{r} 3x^2+4x-5 \\ x-4 \end{array}$$

  2)
$$\begin{array}{r} x^3-x^2+x+1 \\ x-2 \end{array}$$

Procedure. *Multiplying Binomials by the FOIL method.*

Step1.- *Multiply the two* **F***irts terms of each binomial to get the first term of the answer.*
Step2.- *Find the* **O***uter product and the* **I***nner product and add them (mentally, if possible) to get the middle term of the answer.*
Step3.- *Multiply the two* **L***ast terms of the binomials to get the last term of the answer.*

Problem 2. *Using the FOIL method.*

1) $(x-2)(x+1)$

2) $(4x-1)(2x+3)$

Special Products.

Proposition. .

- *Square of a Binomial (Squaring a Binomial).*
$$(x \pm y)^2 = x^2 \pm 2xy + y^2$$
- *Product of the Sum and Difference of Two Terms.*
$$(x+y)(x-y) = x^2 - y^2$$

Problem 3. *Find the product.*

1) $(2x-3)^2$

2) $(6a-3)(6a+3)$

4.7 Division of Polynomials

- Dividing a Polynomial by a Monomial.
To divide a polynomial by a monomial, divide each term of the polynomial by the monomial:
$$\frac{a+b}{c} = \frac{a}{c} + \frac{b}{c} \qquad (c \neq 0)$$

Problem 4. *Divide.*

1) $\dfrac{12m^6 + 18m^5 + 30m^4}{6m^2}$

2) $\dfrac{12p^5 + 8p^4 + 3p^3 - 5p^2}{3p^3}$

- Dividing a Polynomial by a Polynomial.

Problem 5. *Divide.*

1) $\dfrac{p^3 - 2p^2 - 5p + 9}{p+2}$ 2) $(x^3 - 8) \div (x-2)$ 3) $\dfrac{2m^5 + m^4 + 6m^3 - 3m^2 - 18}{m^2+3}$

Instructor: Jorge R. Viramontes
MATHEMATICS 0310

5.1 Greatest Common Factor and Factoring by Grouping.

1) **Factoring out the GCF.**

Definition. Writing an algebraic expression as the product of two or more expression (with degree less than the original one) is called factoring the algebraic expression.
E.g.

$$15x^2 - 6x = 3x(5x - 2)$$

$15x^2 - 6x$ is the algebraic expression and $(3x)(5x - 2)$ is its factored form.
Some expression cannot be factored, we call such expressions **prime expressions**.

Definition. The greatest common factor is the largest term that is a factor of all terms in the polynomial. The greatest common factor is also called the greatest common divisor.

Problem 1. *Factor out the greatest common factor.*

a) $-12p^3q - 8p^2q^2 + 4pq^3$.

b) $9a^4b - 18a^5b + 27a^6b$.

c) $-10x^4y^3z^2 + 8x^3y^2z - 20x^2y$.

2) **Factoring by Grouping.**

Procedure. *Factoring by grouping.*
- *Group the terms so that each group has a common factor*
- *Use the distributive property to factor each group of terms*
- *If possible, factor a common binomial factor from the results of previous step*
- *If step 2 doesn't result in a common binomial factor, try grouping the terms of the original polynomial in a different way.*

Problem 2. *Factor by grouping.*

a) $3ax + 12a + 2bx + 8b$.

b) $16w^4 - 40w^3 - 12w^2 + 30w$.

c) $4a^2b + 12a^2 - 8ab - 24a$.

Instructor: Jorge R. Viramontes
MATHEMATICS 0310

5.2-5.4 Factoring Trinomials.

Procedure. *Factoring Trinomials of the form $x^2 + bx + c$, where $a = 1$.*

- *List all pairs of integers whose product is c*
- *Choose the pair whose sum is equal to b*
- *Suppose that x_1 and x_2 is such pair*
- *Form the product $x^2 + bx + c = (x + x_1)(x + x_2)$*

Problem 1. *Factor the trinomials.*

a) $x^2 - 2x - 15$

b) $x^2 - 9x - 22$

c) $-x^2 - 7xy + 18y^2$.

Procedure. *Factoring Trinomials of the form $ax^2 + bx + c$, where $a \neq 1$.*

- *List all pairs of integers whose product is $a \cdot c$*
- *Choose the pair whose sum is equal to b*
- *Suppose that x_1 and x_2 is such pair*
- *Split the middle term $bx = x_1x + x_2x$, then*
- *$ax^2 + bx + c = ax^2 + x_1x + x_2x + c$, and then factor by grouping*

Problem 2. *Factor the trinomial.*

a) $2x^2 + 7x + 3$.

b) $2x^2 - 8x^3 + 3x$.

c) $10x^2 + 13x - 3$.

Proposition. *Factoring Perfect Square Trinomials.*

$$\boxed{(x \pm y)^2 = x^2 \pm 2xy + y^2}$$

Problem 3. *Factor the perfect square trinomials.*

a) $25x^2 - 30x + 9$.

b) $5w^2 + 50w + 45$.

c) $x^2 + 14x + 49$.

Proposition. *Factoring a Difference of Two Squares.*

$$\boxed{x^2 - y^2 = (x - y)(x + y)}$$

Problem 4. *Factor if possible.*

a) $x^2 - 100$.

b) $4y^4 - 121z^2$.

c) $w^4 - 16$.

5.5 Factoring a Sum and Difference of Two Cubes.

Proposition. *Factoring a Difference or Sum of Two Cubes.*

$$\boxed{x^3 - y^3 = (x - y)(x^2 + xy + y^2) \qquad \textit{and} \qquad x^3 + y^3 = (x + y)(x^2 - xy + y^2)}$$

Problem 1. *Factor if possible.*

a) $27 - 8y^3$.

b) $27 + 8y^3$.

c) $x^3 - 64y^3$.

Instructor: Jorge R. Viramontes
MATHEMATICS 0310

5.6 General Factoring Summary.

Problem 1. *Factor the following expressions.*

a) $abx^2 - 3ax + 5bx - 15.$

b) $d^4 - \frac{1}{16}.$

c) $\frac{1}{9}x^2 + \frac{1}{3}x + \frac{1}{4}.$

d) $(2x-7)^2 - 3(2x-7) - 40.$

e) $4x + 6pa - 8a - 3px.$

f) $x^2 - y^2 - 6y - 9.$